

Convex Optimization and Applications

1 - Introduction

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Organization

■ Contact:

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■ Timing:

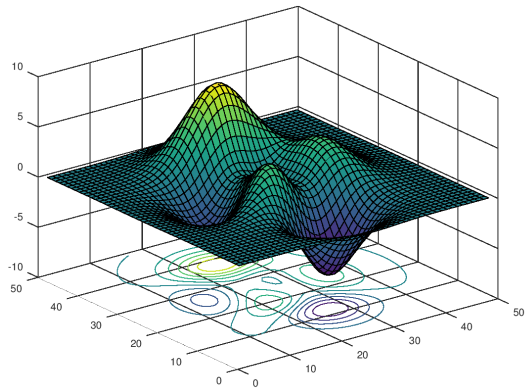
- Monday 10-12, MA841
- Thursday 14-16, MA841
- Lecture every Monday
- Lecture or Exercises every second week on Thursday
- Occasionally: Programming exercises with python notebooks

Resources

- Handout put online as the lecture progresses
<https://www.coga.tu-berlin.de/?197719>
- Slides for selected lectures
- The course is mainly based on the book
“Convex Optimization”,
S. Boyd & L. Vandenberghe, 2004
freely available online at <http://stanford.edu/~boyd/cvxbook/>.
- Selected chapters based on different resources
(cf. website).

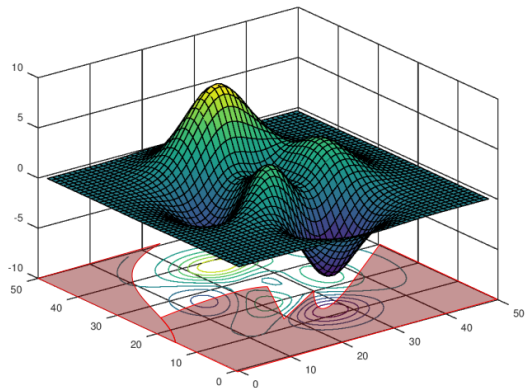
Optimization problem

$$\underset{x \in \mathbb{R}^n}{\text{minimize}} \quad f_0(x)$$



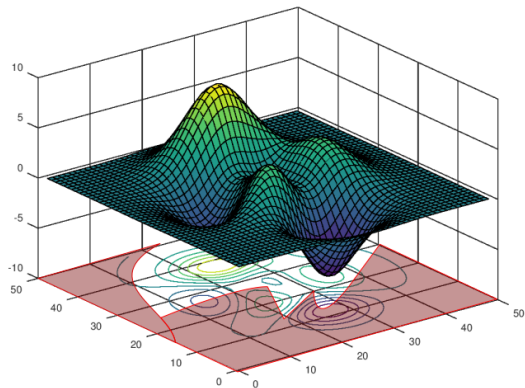
Optimization problem

$$\begin{aligned} & \underset{x \in \mathbb{R}^n}{\text{minimize}} && f_0(x) \\ & \text{s.t.} && f_i(x) \leq 0, \forall i \in [m] \\ & && h_j(x) = 0, \forall j \in [p] \end{aligned}$$



Optimization problem

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- Extremely versatile
- In general, a very hard problem

Optimization problem

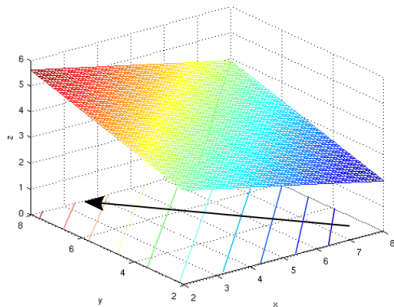
What makes an optimization problem difficult ?

- Number of variables
- Presence of constraints
- Non-convexity \rightarrow Existence of local minima
- Discrete variables
 - This is, in fact, a special case of non-convex constraints:
 - $x_i \in \{0, 1\} \iff x_i^2 = x_i.$
 - $x_i \in \mathbb{Z} \iff \sin \frac{x_i}{\pi} = 0.$
- Regularity (smoothness) of functions.
 - existence of first or second-order derivatives, subgradients, ...

When everything is linear

Linear Programming

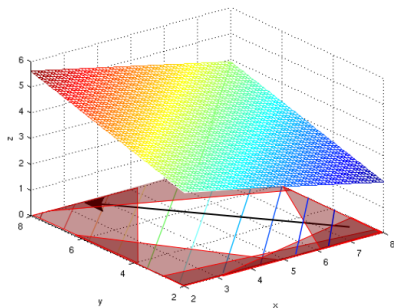
$$\underset{x \in \mathbb{R}^n}{\text{minimize}} \quad \langle c, x \rangle$$



When everything is linear

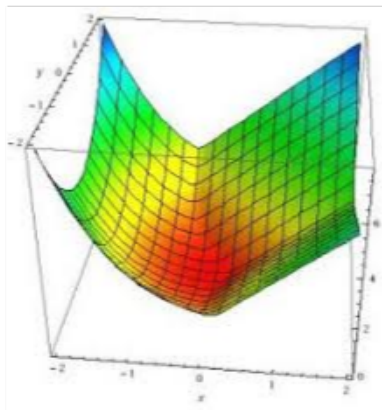
Linear Programming

$$\begin{aligned} & \underset{x \in \mathbb{R}^n}{\text{minimize}} && \langle c, x \rangle \\ & \text{s.t.} && \langle a_i, x \rangle \leq 0, \forall i \in [m] \\ & && \langle a_j, x \rangle = 0, \forall j \in [p] \end{aligned}$$



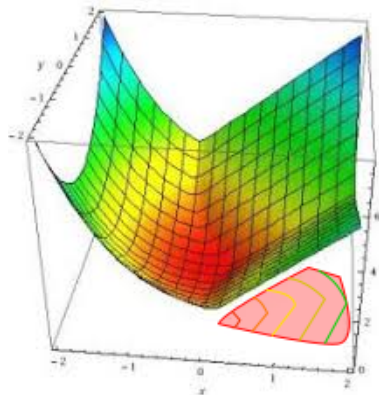
Convex optimization

$$\underset{x \in \mathbb{R}^n}{\text{minimize}} \quad f_0(x)$$



Convex optimization

$$\begin{aligned} & \underset{\mathbf{x} \in \mathbb{R}^n}{\text{minimize}} && f_0(\mathbf{x}) \\ & \text{s.t.} && f_i(\mathbf{x}) \leq 0, \forall i \in [m] \\ & && h_j(\mathbf{x}) = 0, \forall j \in [p] \end{aligned}$$



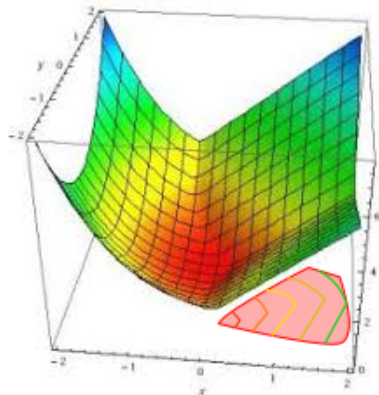
Convex optimization

As a nonlinear program:

$$\begin{aligned} & \underset{x \in \mathbb{R}^n}{\text{minimize}} && f_0(x) \\ & \text{s.t.} && f_i(x) \leq 0, \forall i \in [m] \\ & && h_j(x) = 0, \forall j \in [p] \end{aligned}$$

As a conic program:

$$\begin{aligned} & \underset{x \in \mathbb{R}^n}{\text{minimize}} && \langle c, x \rangle \\ & \text{s.t.} && Ax = b \\ & && x \in K \end{aligned}$$



Why studying convex optimization ?

- Probably largest class of efficiently solvable problems
- Many applications ! Power of representation \gg LP
- “Locally, everthing is convex”
 - For many applications, finding a good local optimum is enough
 - This is often done by using convex optimization algos
- Convex relaxations of non-convex problems
 - Obtain lower bounds
 - Sometimes, rounding algorithms with a provable approximation guarantee

Objectives of this course

- Standard nonlinear programming approach vs. “modern” conic programming approach.
- Understand the concept of duality in convex optimization.
- Modelling issues: what can be (re)formulated as a convex optimization problem ?
- Review many applications of convex optimization
- Historical developments
- Learn to use modern python interfaces to solve conic problems with state-of-the art solvers in python
- Overview of Algorithms to solve convex problems
- Convex relaxations of non-convex (in particular, combinatorial) optimization problems

Evaluation

- Exercises
 - given 1 week in advance
 - check the exercises you've prepared. One student will be asked to explain his/her solution
 - You need 50% of all exercises *checked* to take the exam. (6-7 sessions, 2-3 ex./session → at least approx. 9 exercises)
- Oral examination.
 - Do not learn everything *by heart*.
 - However, you should know important definitions and have a rough idea of important proofs
 - You will not be asked to prove results from the courses.
 - Know which results exist and be able to find them in your handout if needed.
 - The focus is more on your general understanding, and your ability to solve new exercises