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ADM II
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10. Exercise sheet

Due-date: Friday, 27.06.2014, *before* the lecture has started

Exercise 54 **2+3+5 points**

Consider the integer programming problem

$$\begin{array}{lrcrcr} \text{maximize} & x_1 + 2x_2 & & & & \\ \text{subject to} & -3x_1 & + & 4x_2 & \leq & 4 \\ & 3x_1 & + & 2x_2 & \leq & 11 \\ & 2x_1 & - & x_2 & \leq & 5 \\ & & & x_1, x_2 & \geq 0 & \\ & & & x_1, x_2 & \text{integer} & \end{array}$$

Sketch the feasible set and use the figure to answer the following questions:

(a) What is the optimal cost of the linear programming relaxation? What is the optimal cost of the integer programming problem and how large is the gap between the two values?

(b) What is the convex hull of the feasible set of the integer program. Give the inequalities that forms the polyhedra.

(c) Compute and illustrate the first Chvátal closure in this example. What is the optimal solution of the new linear program?

Exercise 55 **5 points**

Use the following example to show that the guaranteed approximation factor $\frac{3}{2}$ of Christofides' algorithm cannot be decreased:
Let $V = \{v_1, \ldots, v_n\}$ and define $c_{v_i v_j} := \lfloor \frac{|i-j|+1}{2} \rfloor$ for $i \neq j$

Exercise 56 **5 points**

This exercise demonstrates that it is important to have a rational polyhedron in order to deal with the convex hull of the integral points.

(a) Let $P := \{(y, x) \in \mathbb{R}^2 \colon x \leq \sqrt{2}y\}$. Show that the convex hull of the integer points in P, i.e., P_I, does not form a polyhedron.

(b) Let $P := \{(y, x) \in \mathbb{R}^2 \colon x \leq \sqrt{2}y, y \geq 0\}$. Show that $P^{(t)} = P \neq P_I$.

Exercise 57 **5 bonus points**

The following statement describes an easy construction for polytopes with big Chvátal rank: Let P be the convex hull of the three points $(0, 0)$, $(0, 1)$, $(k, \frac{1}{2})$ in $\mathbb{R}^2$.

Prove that $P^{(2k-1)} \neq P_I = P^{(2k)}$.

Exercise 58 **Tutorial session – 0 points**

Let P =conv. hull$(\{(0,0),(1,0),(\frac{1}{2},3)\})$. Find a system of linear inequalities that defines $P' = \{x \in P \mid x \text{ satisfies every GC-cut for } P\}$.

Exercise 59 **Tutorial session – 0 points**

A variant on the TSP permits a node to be visited more than once if it results in a better solution. Show that if the cost function satisfies the triangle-inequality, then there is always an optimum solution which visits each node exactly once.

Exercise 60 **Tutorial session – 0 points**

Let K_n be the complete undirected graph with n nodes. Prove the following lemma which can be used to show that the polytope which describes the convex hull of all cycles in the graph has a dimension of exactly $\frac{n(n-3)}{2}$:

For every $k \geq 1$:

(a) The edge set of K_{2k+1} can be partitioned into k edge disjoint cycles.

(b) The edge set of K_{2k} can be partitioned into $k-1$ edge disjoint cycles and a perfect matching.