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ADM II
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9. Exercise sheet

Due-date: Friday, 20.06.2014, *before* the lecture has started

Exercise 46 **5 points**

Give an example of a matrix $A \in \{-1, 0, 1\}^{m \times n}$ and a vector $b \in \mathbb{Z}^m$ such that $\{x \mid Ax \leq b,\ x \geq 0\}$ is an integral polytope but A is not TUM.

Exercise 47 **5 points**

Let $G = (V, E)$ be an undirected graph and consider the perfect matching polytope P defined by the inequalities

$$x(\delta(v)) = 1 \text{ for all } v \in V \tag{1}$$

$$x(\delta(U)) \geq 1 \text{ for all } U \subseteq V \text{ with } 3 \leq |U| \text{ odd.} \tag{2}$$

$$x_e \geq 0 \text{ for all } e \in E. \tag{3}$$

Given any subset $U \subseteq V$ with $3 \leq |U|$ odd, give a cutting-plane proof of $x(E(U)) \leq \frac{|U|-1}{2}$ starting from system (1)-(3).

Exercise 48 **5 points**

Let $G = (V, E)$ be an undirected graph and $A \in \{0, 1\}^{V \times E}$ its vertex-edge incidence matrix. Show that A is TUM if and only if G is bipartite.

Exercise 49 **5 points**

A matrix A with entries in $\{0, 1\}$ is said to have the *consecutive-ones-property* if the nonzero entries in each column occur consecutively. Show that such a matrix is TUM.

Exercise 50 **Tutorial session – 0 points**

Let $A \in \mathbb{R}^{m \times n}$ be TUM.

(a) Convince yourself that also $-A$, $[A \; -A], [A \; I_m], A^T$, all submatrices of A, and all matrices obtained by a sequence of column- and/or row-permutations on A are TUM. (As usual, I_m is the m by m identity matrix.)

(b) Given $b \in \mathbb{Z}^m$ and $l, u \in \mathbb{Z}^n$, show that the polyhedra

- $\{x \in \mathbb{R}^n \mid Ax = b, \; x \geq 0\}$,
- $\{x \in \mathbb{R}n \mid Ax \leq b\}$,
- $\{x \in \mathbb{R}^n \mid Ax \leq b, \; x \geq 0\}$,
- $\{x \in \mathbb{R}^n \mid Ax \leq b, \; l \leq x \leq u\}$, and
- $\{x \in \mathbb{R}^n \mid Ax = b, \; l \leq x \leq u\}$

are integral.

Exercise 51 **Tutorial session – 0 points**

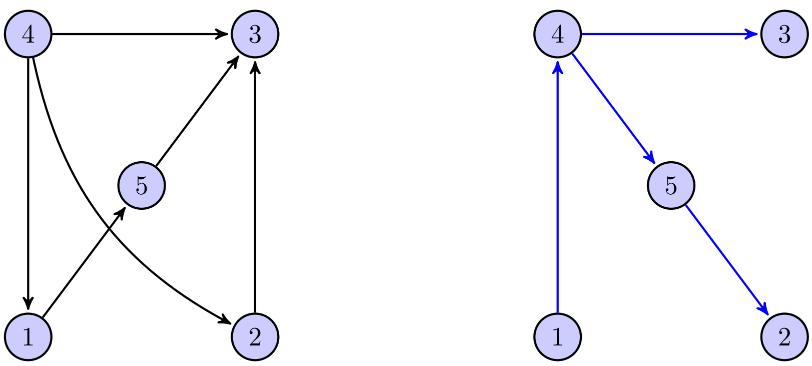
Let $A \in \mathbb{R}^{m \times n}$.

a) Show that A is TUM if and only if $[A \; I_m]$ is unimodular.

b) Let A have full row rank and B be a basis of A. Prove that $B^{-1}A$ is TUM.

Exercise 52 **Tutorial session – 0 points**

Consider the digraph $D = (V, E)$ and the spanning tree $T = (V, E')$ in the figure below.



$D = (V, E)$ $T = (V, E')$

(a) Write down the vertex-edge incidence matrix $A \in \{-1, 0, 1\}^{V \times E}$ of D.

(b) Write down the network matrix w.r.t. D and T.

(c) Draw a tree $\tilde{T}$ so that A is the network matrix w.r.t. D and $\tilde{T}$.

Exercise 53 **Tutorial session – 0 points**

Use total unimodularity to prove the MAX-FLOW-MIN-CUT Theorem.