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ADM II
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8. Exercise sheet

Due-date: Friday, 13.06.2014, *before* the lecture has started

Exercise 42 **2+2+2 points**

An *integer linear program (ILP)* is a linear program with the additional constraints that all variables should be integral. Formulate the following combinatorial optimization problems as ILPs:

(a) Given a knapsack of capacity b and n items. Each item i has a certain weight w_i as well as a value v_i indicating how worthful it is to have this item packed into the knapsack. The task is to select a subset of items of maximum value so that that the total weight of the selected items does not exceed the capacity b.

(b) Given a directed graph $G = (V, A)$, two vertices $s, t \in V$ and arc capacities $u : A \to \mathbb{Z}$, find an integral s, t-flow of maximum value.

What can you say about the complexity of the problems?

Exercise 43 **5 points**

Show that the problem to find the greatest common divisor of two integers $a, b \in \mathbb{Z}$ can be formulated as the following ILP:

$$\gcd(a, b) = \min\{ax + by \mid ax + by \geq 1,\ x, y \in \mathbb{Z}\}.$$

Exercise 44 **5 points**

Let $G = (V, E)$ be a bipartite graph. Use Lemma 14.9 to show that all vertices of the polytope $P := \{x \in \mathbb{R}^E_+ \mid x(\delta(v)) = 1\ \forall v \in V\}$ are integral.

Exercise 45 **4 points**

Let $G = (V, E)$ be a (not necessary bipartite) graph. Show that a vector $x \in \mathbb{R}^E$ is a vertex of $P := \{x \in \mathbb{R}^E_+ \mid x(\delta(v)) = 1\ \forall v \in V\}$ if and only if $x \in \{0, \frac{1}{2}, 1\}^E$ and the edge set $E^x := \{e \in E \mid x_e \notin \mathbb{Z}\}$ form vertex disjoint odd circuits.