Technische Universität Berlin
Institut für Mathematik

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ADM II
Summer Term 2014

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1. Exercise sheet

Due-date: Friday, 25.04.2014, *before* the lecture has started

Exercise 0 — -5 ∨ 0 points

This exercise is obligatory!

Each member of your group need to send an email to **combi-adm2@math.tu-berlin.de** containing her/his personal information. The line has to be in the following format: (without changing the order of entries, additional spaces, etc.):

matriculation number, ␣ *lastname*, ␣ *firstname*, ␣ *study course*, ␣ *account number*, ␣ *gender*, ␣ *email*

The following lines are the possible content of three separate emails:

```
123456, M\"uller, Lieschen, TWM, 108, f, liesel@gmx.net

654321, Klein, H\"anschen,  ITM, 108, m, hansi@dot.com

987654, Strau\"s, Andr\'e, Mathe Diplom, 108, m, andre@strauss.de
```

The information is used to hand out certificates for this course. You might want to avoid typos.

Exercise 1 — 5 points

Use the Fourier-Motzkin-Elimination to determine whether or not the following system of linear inequalities has a solution (successively eliminate the variables x_1, x_2, and x_3):

$$\begin{aligned} 2x_1 + 6x_2 + 2x_3 - x_4 &\leq 1 \\ x_1 - 2x_2 + 3x_3 + x_4 &\leq 0 \\ -3x_1 + 6x_2 - 2x_3 - 3x_4 &\leq 2 \\ -x_1 - 3x_2 - 3x_3 - 3x_4 &\leq -3 \\ 3x_2 + x_3 + x_4 &\leq 2 \end{aligned}$$

Exercise 2 — 6 points

Consider two non-empty polyhedra $P := \{x \in \mathbb{R}^n \mid Ax \leq b\}$ and $Q := \{x \in \mathbb{R}^n \mid Dx \leq d\}$. Show that $P \cap Q = \emptyset$ if and only if there is a vector $c \in \mathbb{R}^n$ such that $c^T x < c^T y$ for all $x \in P$ and $y \in Q$.

Hint: Formulate the problem of finding a point in $P \cap Q$ as a linear program. Consider the dual of this linear program.

Exercise 3 — 4 points

Show that element x of a polytope $P \subset \mathbb{R}^n$ can be expressed as a convex combination of at most $n + 1$ extreme points of P.

Hint: Consider an extreme point of the set of all possible representations of x.

Remark: This result is known as *Carathéodory's Theorem.*