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Modulklausur
Introduction to Linear and Combinatorial Optimization

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Firstname	
Lastname	
Matriculation Number	

- Please fill in the cover sheet and **wait for the signal to start.**
- The examination takes **120 minutes**.
- At least **30 points** needed to pass the exam.
- **Write your name on every single sheet of paper!**
- You can ask for **additional sheets** of paper.
- All sheets need to stay **stapled**.
- **Using items** like scripts, notes or electronic devices (including cellphones) is **not allowed** and will be treated as an attempt to defraud.
- Red pens are forbidden, answers not written with permanent ink will not be graded.
- Select **one topic** that will not be graded.

We wish you good luck!

Please **select the canceled section** and leave all other all boxes blank.

Exercise	1	2	3	4	5	Σ
maximum	15	15	15	15	15	60
achieved						
corrected						
canceled	○	○	○	○	○	

Topic 1: LP Basics & Geometry of LPs

Topic 2: Simplex Method & Duality

Exercise 1 (5 Points)

Define the diameter $D(P)$ of a polyhedron P and the functions $\triangle(n,m)$ and $\triangle_u(n,m)$.

Let $P_1 := \{x \in \mathbb{R}^2 \mid 1 \leq |x_i| \leq 3,\ x_1 + x_2 \leq 5,\ x_1 - x_2 \leq 2\}$.

- What is the value of $D(P_1)$?
- What is the value of $\triangle(2,17)$?
- What is the value of $\triangle_u(1,9)$?

Exercise 2 (5 Points)

With phase I of the simplex algorithm, one can find a basic feasible solution for the following LP:

$$\begin{aligned} \min\ & 2x + 3y - z \\ & x - 2y \leqslant 4 \\ & y + z \geqslant 2 \\ & x + y + z = 3 \\ & x, y, z \geqslant 0 \end{aligned}$$

Use the full-tableau method and introduce artificial variables only when necessary. Start to write down the auxiliary problem in standard form with slack variables for the first two constraints and two additional artificial variables.

Perform a single pivot operation to remove an artificial variable from the basis. Use the first possible variable, list the pivot element and the resulting basis.

The resulting tableau of phase I is:

	x	y	z	s_1	s_2	a_1	a_2
0	0	0	0	0	0	1	1
3	0	-2	0	1	-1	1	-1
2	0	1	1	0	-1	1	0
1	1	0	0	0	1	-1	1

What is the resulting solution for the original problem? Is this solution optimal?

Exercise 3 (5 Points)

Consider the following pair of primal and dual LPs:

$$\begin{aligned} \min\ & c^T \cdot x \\ \text{s.t.}\ \ & A \cdot x \ \geq \ b \end{aligned} \qquad\qquad \begin{aligned} \max\ & p^T \cdot b \\ \text{s.t.}\ \ & p^T \cdot A \ = \ c^T \\ & p \geq 0 \end{aligned}$$

Formulate and prove the Complementary Slackness Theorem. You can use the Weak Duality Theorem/Strong Duality Theorem for your proof.

Topic 3: Optimal Trees & Paths

Topic 4: Maximal & Min Cost Flows

Topic 5: NP-Completeness