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ADM I
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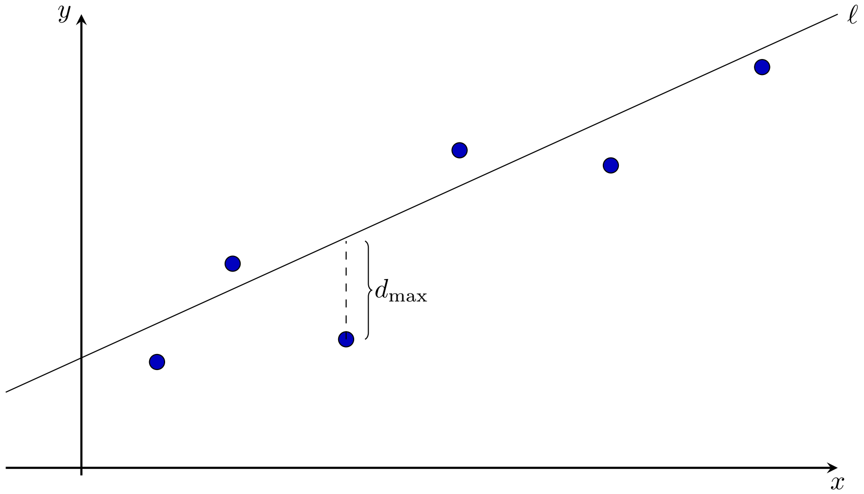
6. Exercise sheet

Due-date: Friday, 29.11.2013, *before* the exercise session has started

Exercise 27 **3+2+1 points**

Assume that a physicist takes n measurements of a variable which can be thought of as n points $(x_1, y_1), \ldots, (x_n, y_n)$ in a planar coordinate system. The physicist wants to find a line ℓ with the equation $y = ax + b$, that best fits the observed data, that is he wants to minimize the largest vertical distance $d_{\max}$ from ℓ to any point (x_i, y_i).



(a) Set up a linear program that models the problem.

(b) Find the dual program to the one above.

(c) Which one of the two programs would you solve if you had to solve it by hand and why?

Exercise 28 **4 points**

Suppose we have a subroutine which, given a system of linear inequality constraints, either produces a solution or decides that no solution exists. Describe a procedure that uses a single call to this subroutine to find an optimal solution to any linear programming problem that has an optimal solution.

(This shows that essentially solving linear programming problems is not harder than solving systems of linear inequalities. That is, finding a feasible LP solution is as difficult as finding an optimal LP solution.)

Exercise 29 **5 points**

Let A be a given matrix. Show that exactly one of the following alternatives must hold.

(a) There exists some $x \neq 0$ such that $Ax = 0, x \geq 0$.

(b) There exists some y such that $y^T A > 0$.

Exercise 30 **5 points**

Find the dual linear program for $\min\{c_1 x + c_2 y \mid A_1 x + A_2 y \geq d_1, B_1 x + B_2 y = d_2, x \geq 0\}$.

Exercise 31 **Tutorial session – 0 points**

Give examples for linear programs that satisfy the following conditions.

(a) The primal problem has a degenerate optimal basic feasible solution and the dual has a unique optimal solution.

(b) Both the primal and the dual problem have more than one optimal solution.

(c) Both the primal and the dual problem have no feasible solution.

Exercise 32 **Tutorial session – 0 points**

Decide with the conditions of complementary slackness whether $\mathbf{x}^* = (0, \frac{4}{3}, \frac{2}{3}, \frac{5}{3}, 0)^\top$ is an optimal solution to the following linear program.

$$\begin{array}{ll} \text{maximize} & 7x_1 + 6x_2 + 5x_3 - 2x_4 + 3x_5 \\ \text{subject to} & \begin{array}{rcrcrcrcrcl} x_1 & + & 3x_2 & + & 5x_3 & - & 2x_4 & + & 2x_5 & \leq & 4 \\ 4x_1 & + & 2x_2 & - & 2x_3 & + & x_4 & + & x_5 & \leq & 3 \\ 2x_1 & + & 4x_2 & + & 4x_3 & - & 2x_4 & + & 5x_5 & \leq & 5 \\ 3x_1 & + & x_2 & + & 2x_3 & - & x_4 & - & 2x_5 & \leq & 1 \\ & & & & & & \multicolumn{3}{r}{x_1, x_2, x_3, x_4, x_5} & \geq & 0 \end{array} \end{array}$$

Exercise 33 **Tutorial session – 0 points**

What is wrong with the following argument:

By duality and the fact that $\min\{f(x) : x \in X\} \leq \max\{f(x) : x \in X\}$ we have

$$\begin{aligned} \max\{\, \mathbf{c}^\top\mathbf{x} \mid \mathbf{A}\mathbf{x} \leq \mathbf{b},\ \mathbf{x} \geq \mathbf{0} \,\} &\leq \min\{\, \mathbf{p}^\top\mathbf{b} \mid \mathbf{p}^\top\mathbf{A} \geq \mathbf{c}^\top,\ \mathbf{p} \geq \mathbf{0} \,\} \\ &\leq \max\{\, \mathbf{p}^\top\mathbf{b} \mid \mathbf{p}^\top\mathbf{A} \geq \mathbf{c}^\top,\ \mathbf{p} \geq \mathbf{0} \,\} \\ &\leq \min\{\, \mathbf{c}^\top\mathbf{x} \mid \mathbf{A}\mathbf{x} \geq \mathbf{b},\ \mathbf{x} \leq \mathbf{0} \,\} \\ &\leq \max\{\, \mathbf{c}^\top\mathbf{x} \mid \mathbf{A}\mathbf{x} \geq \mathbf{b},\ \mathbf{x} \leq \mathbf{0} \,\} \\ &\leq \min\{\, \mathbf{p}^\top\mathbf{b} \mid \mathbf{p}^\top\mathbf{A} \leq \mathbf{c}^\top,\ \mathbf{p} \leq \mathbf{0} \,\} \\ &\leq \max\{\, \mathbf{p}^\top\mathbf{b} \mid \mathbf{p}^\top\mathbf{A} \leq \mathbf{c}^\top,\ \mathbf{p} \leq \mathbf{0} \,\} \\ &\leq \min\{\, \mathbf{c}^\top\mathbf{x} \mid \mathbf{A}\mathbf{x} \leq \mathbf{b},\ \mathbf{x} \geq \mathbf{0} \,\} \\ &\leq \max\{\, \mathbf{c}^\top\mathbf{x} \mid \mathbf{A}\mathbf{x} \leq \mathbf{b},\ \mathbf{x} \geq \mathbf{0} \,\}. \end{aligned}$$

Therefore every inequality holds with equality and there is no difference between minimizing and maximizing in a linear program.