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ADM I
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3. Exercise sheet

Due-date: Friday, 08.11.2013, *before* the exercise session has started

Exercise 12 — 4 points

Model the problem described in Exercise 3 using *ZIMPL* . Do not restrict, however, your model to a fixed number of products. This means the model should be able to handle an arbitrary number of products.

The input data of the model should be given from the outside. Therefore, you find on our webpage three data files (`data1`, `data2`, `data3`). An example is given below:

```
hours 20000
money 4400
reuse 40
bourbon  5   3 3
whiskey  4.5 2 4
```

The first line gives the total number of available machine hours. The second line states the available working capital. The third line gives the percentage of the sale revenues which are made available to finance the ongoing operations. Additionally, the file contains one line for each product. The first column states the name of the product, column two the sales price, the third column the production cost and the last column the required machine hours to produce this product.

Create a file named `cli.zpl` which contains the *ZIMPL* model. **The first line of this file has to start with a comment line, stating your group number.** The model expects a data file with the name `FILE`, parses the necessary information from the given file, and creates the corresponding linear program. As example, the following command should generate the lp file for the data set `data1`:

```
zimpl -DFILE=data1 cli.zpl
```

Send your *ZIMPL* file to Torsten — gellert@math.tu-berlin.de

Exercise 13 — 3+3 points

List all basic solutions and their basis for the given polyhedra. Which of these are basic feasible solutions?

(a) $P = \{(x, y, z) \in \mathbb{R}^3 \mid 3x + y + z = 6,\ z \leq 3,\ (x, y, z) \geq 0\}$

(b) $P = \{(x, y, z) \in \mathbb{R}^3 \mid x + y - z = -3,\ 2x + y + z = 4,\ (x, y, z) \geq 0\}$

Exercise 14 — 5+5 points

Consider the polyhedron decribed by the following constraints.

$$\begin{array}{rcrcrcr}
-3x_1 & & & + & 2x_3 & \leq & -2 \\
3x_1 & + & 6x_2 & + & 2x_3 & \leq & 10 \\
12x_1 & + & 3x_2 & + & x_3 & \leq & 26 \\
-6x_1 & - & 9x_2 & + & 7x_3 & \leq & 2 \\
x_1 & - & x_2 & - & 2x_3 & \leq & 8 \\
2x_1 & + & x_2 & - & 2x_3 & \leq & 20
\end{array}$$

Show that the points $(-8, 10, -13)^\top$ and $(2, 0, 2)^\top$ are both degenerate basic feasible solutions. In either case explain the respective reason for the degeneracy and whether you can actually remove it without changing the feasible set.

Exercise 15 **Tutorial session – 0 points**

Consider the polyhedron $P := \{x \in \mathbb{R}^2 \mid A \cdot x \leq b\}$ given by:

$$A = \begin{pmatrix} -1 & -1 \\ 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad b = \begin{pmatrix} -1 \\ 1 \\ 2 \\ 0 \\ 0 \end{pmatrix} .$$

(a) Give a graphical representation of the polyhedron and write down all vertices.

(b) Transform the corresponding LP $\min\{c^T \cdot x \mid A \cdot x \leq b\}$, with $c = (c_1, c_2)^T \in \mathbb{R}^2$, into an equivalent LP in standard form. Write down all basis matrices of the LP in standard form.

(c) Give all basic solutions of the LP in standard form. Which of them are degenerate and which are feasible? Identify the corresponding vertices of the polyhedron P.

Exercise 16 **Tutorial session – 0 points**

Consider the linear optimization problem

$$\begin{array}{lrcrcrcr} \text{minimize} & x_1 + 2x_2 + x_3 & & & & & & \\ \text{subject to} & -6x_1 & + & 9x_2 & + & x_3 & \leq & 21 \\ & -2x_1 & - & x_2 & - & x_3 & \leq & 3 \\ & & & -3x_2 & - & x_3 & \leq & 3 \\ & 9x_1 & + & 6x_2 & + & 5x_3 & \leq & 27 \\ & x_1 & - & x_2 & & & \leq & 3 \end{array}$$

Show that the set of feasible solutions is unbounded. Can you still find an optimal solution?