Algorithmic Game Theory — Assignment Sheet 6

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January 20, 2016

Due: January 27, 2016

Exercise 6.1. The United Nations Security Council has fifteen members, five of which are permanent. Resolutions require the positive vote of at least nine members to pass, but can be vetoed by any permanent member. Define a coalitional game (N, v) such that $v(S) = 1$ if a resolution supported by the members of $S \subseteq N$ passes and $v(S) = 0$, otherwise. Compute the Shapley value of the game.

Exercise 6.2. A coalitional game is called convex if the characteristic function is supermodular, i.e., $v(S) + v(T) \leq v(S \cup T) + v(S \cap T)$ for all $S, T \in 2^N$. Prove that in every convex game, the vector of Shapley values is contained in the core.

Exercise 6.3. Give an example of a game with a non-empty core where the vector of Shapley values is not in the core.

Exercise 6.4. Consider a congestion game where costs are set-dependent and shared according to the Shapley value among its users. Specifically, each resource $e \in E$ is associated with a cost function $c_e : 2^N \to \mathbb{R}$, and the set of strategies of player i is $S_i \subseteq 2^E$. The private cost of player i in strategy profile $\mathbf{s}$ is defined as

$$\pi_i(\mathbf{s}) = \sum_{e \in s_i} \phi_i(c_e|_{2^{N_e(\mathbf{s})}}),$$

where $(\phi_i)_{i \in N}$ is the Shapley value, $N_e(\mathbf{s}) = \{i \in N : e \in s_i\}$ is the set of users of resource e under strategy profile $\mathbf{s}$, and $c_e|_{2^{N_e(\mathbf{s})}}$ is the restriction of c_e to $2^{N_e(\mathbf{s})}$. Show that congestion games with Shapley cost sharing are exact potential games.

Exercise 6.5. A threshold game is a congestion game with resources $E = E_{\text{out}} \cup E_{\text{in}}$ where $E_{\text{out}} = \bigcup_{i \in N}\{e_i\}$ and $E_{\text{in}} = \bigcup_{i,j \in N, i \neq j}\{e_{ij}\}$. Each player i has two strategies, $S_i = \{s_i^{\text{out}}, s_i^{\text{in}}\}$ where $s_i^{\text{out}} = \{e_i\}$ and $s_i^{\text{in}} = \big\{e_{ij} : j \in N \setminus \{i\}\big\}$. Show that computing a pure Nash equilibrium in a threshold game with linear cost functions is PLS-complete.

Exercise 6.6. A market sharing game is a congestion game where each resource is associated with a profit q_e and a size l_e. Each player i is associated with a subset $E_i \subset E$ of allowable resources and a budget b_i. The players' strategies are defined as $S_i = \{F \subseteq E_i : \sum_{e \in F} l_e \leq b_i\}$ and the private cost is $\pi_i(\mathbf{s}) = -\sum_{e \in s_i} q_e / x_e(\mathbf{s})$ where $x_e(\mathbf{s})$ is the number of players using e in $\mathbf{s}$. Show that it is PLS-compute to compute a pure Nash equilibrium of a market sharing game, even if all l_e are polynomial. You may use the result proven in Exercise 6.5. Then, argue that a pure Nash equilibrium can be computed in polynomial time, if $l_e = 1$ for all $e \in E$.