Algorithmic Game Theory — Assignment Sheet 5

M. Klimm

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Due: January 6, 2016

Exercise 5.1. For a finite set of resources E with cost functions $(c_e)_{e \in E}$, an allocation vector $\mathcal{A}$ and a vector of resource-dependent demands $(d_{i,e})_{i \in N, e \in E}$, the corresponding congestion game with resource dependent demands is the strategic game G with $S_i = A_i$ and $\pi_i(\mathbf{s}) = \sum_{e \in s_i} d_{i,r} c_e(x_e(\mathbf{s}))$ for all players i, where $x_e(\mathbf{s}) = \sum_{j \in N : e \in s_j} d_{j,e}$. Show that congestion games with resource-dependent demands and affine costs have a pure Nash equilibrium.

Exercise 5.2. A congestion game is called a singleton game if $|s_i| = 1$ for all $i \in N$ and $s_i \in S_i$. Show that singleton weighted congestion games with strictly increasing cost functions have a pure Nash equilibrium by showing that the vector that contains the players' private costs (not multiplied with d_i) sorted in non-increasing order decreases lexicographically along any sequence of unilateral improvements.

Exercise 5.3. Show that if a set $\mathcal{C}$ of strictly increasing and continuous cost functions is consistent for <u>two-player</u> weighted congestion games then for each two functions $f, g \in \mathcal{C}$ there are $a, b \in \mathbb{R}$ such that $f(x) = ag(x) + b$ for all $x \geq 0$. Here, a set of cost functions is consistent if all two-player weighted congestion games with costs out of the set has a pure Nash equilibrium.

Exercise 5.4. Show that for a continuous and non-monotonic function $c : \mathbb{R}_{\geq 0} \to \mathbb{R}$, there are $x, y \in \mathbb{R}_{>0}$ such that either $c(x) < c(x+y) < c(y)$ or $c(x) > c(x+y) > c(y)$. Use this result to argue that every set of cost functions that is consistent for singleton weighted congestion games only contains monotonic functions.

Exercise 5.5. Show that for unweighted network congestion games on directed graphs, the problem of deciding whether there is a Nash equilibrium with social cost less than k is NP-complete.

Exercise 5.6. Give a winning strategy for player 1 for the Poisoned-Chocolate game played on a square chocolate of size $n \times n$.

There are no lectures on 15.12.2015 and 16.12.2015. The lecture on 05.01.2016 and the exercise session on 06.01.2016 will be held by Felix Fischer. The lecture on 12.01.2016 will not take place. Instead, the lecture on 13.01.2016 will be from 14:15 to 17:30.

Assignment sheet 6 is the last regular assignment sheet and will be published on January 13, 2016 and due on January 20, 2016. There will be an extra assignment sheet to be published on January 27, 2016 and due on February 3, 2016. You only need to solve 24 exercises of all seven assignements in total.

Merry christmas and a happy new year!