Algorithmic Game Theory — Assignment Sheet 3

M. Klimm

November 18, 2015

Due: November 25, 2015

Exercise 3.1. The centipede game with n periods is a two-player game with game tree $T = (V, E)$ where

$$V = \{v_k : k \in \{0, \dots, n\}\} \cup \{\bar{v}_k : k \in \{0, \dots, n-1\}\},$$
$$E = \{(v_{k-1}, v_k) : k \in \{1, \dots, n\}\} \cup \{(v_k, \bar{v}_k) : k \in \{0, \dots, n-1\}\}.$$

The decision vertices are $V_1 = \{v_k : k \text{ even}\}$ and $V_2 = \{v_k : k \text{ odd}\}$, and leaf utilities are $\hat{u}(\bar{v}_k) = (k+1, k-1)$ for k even, $\hat{u}(\bar{v}_k) = (k-1, k+1)$ for k odd, and $\hat{u}(v_n) = (n, n-1)$. Determine all subgame-perfect equilibria.

Exercise 3.2. Give two examples of extensive games with perfect information that do not have a pure Nash equilibrium, one where the game tree has finite height, and one where the game tree has a finite maximum degree.

Exercise 3.3. Define an extensive version of the Cournot Oligopoly discussed in the lecture in which player 1 moves first. In the Cournot Oligopoly, two firms produce a homogenous good without cost and sell the produced quantities on a single market with market reaction function $p(x) = \max\{0, 1 - x\}$ for all $x \geq 0$. Compute the subgame perfect equilibrium and discuss the difference to the simultaneous moves game.

Exercise 3.4. Two players fight a duel. They start $2n + 1$ paces apart from each other, with a single bullet each. In every round, they simultaneously advance one pace and decide whether to fire a shot, or not. The probability of hitting the opponent in round i is i/n. The game ends after n rounds, or when a player is hit, whichever happens earlier. If exactly one player is hit, this player receives utility -1 while the other gets utility 1. In all other cases, both get 0. The guns are silent, so neither player knows before the end of game whether the other has fired a shot, or not. Show that firing a shot in round 2 is optimal for $n = 4$, while the mixed strategy in which a shot in round i is fired with probability 0 if $i \in \{1, 4\}$, probability $\frac{5}{11}$ if $i \in \{2, 3\}$, and probability $\frac{1}{11}$ if $i = 5$ is optimal for $n = 5$.

Exercise 3.5. An extensive game with exogenous uncertainty is an extensive game in which all information sets are singletons and an artificial player 0 who does not receive any utility plays according to a fixed behavioral strategy. Prove that finite extensive games with endogenous uncertainty have a pure Nash equilibrium.

Exercise 3.6. Consider the variant of Tic-Tac-Toe played on the infinite square grid where a player wins when marking 9 consecutive cells in a horizontal, vertical, or diagonal line with its symbol. Prove that both players can guarantee a draw.