

Exercise 1.1. Show that the mixed extension of the three-player game below with $S_1 = \{\text{Top}, \text{Bottom}\}$, $S_2 = \{\text{Left}, \text{Right}\}$ and $S_3 = \{\text{Front}, \text{Back}\}$ has a unique Nash equilibrium in which all players play according to an irrational strategy.

	Left	Right		Left	Right
Top	(3, 0, 2)	(0, 2, 0)	Top	(1, 0, 0)	(0, 1, 0)
Bottom	(0, 1, 0)	(1, 0, 0)	Bottom	(0, 1, 0)	(2, 0, 3)
	Front			Back	

Exercise 1.2. Determine all Nash equilibria of the mixed extension of the Hawk-Dove-Game below, depending on the parameters $a < b < c < d$.

	Hawk	Dove
Hawk	(c, c)	(b, d)
Dove	(d, b)	(a, a)

Exercise 1.3. Let \mathcal{G} be the class of mixed extensions of finite games with two players and $|S_1| = |S_2| = 2$. Determine the set

$$\mathcal{N} = \{n \in \mathbb{N} \cup \{0, \infty\} : \text{there is } G \in \mathcal{G} \text{ with exactly } n \text{ Nash equilibria}\}.$$

Exercise 1.4. Let G be a two-player zero-sum game with compact strategy sets S_i for $i = 1, 2$ and a continuous utility function u_1 . Prove that $\sigma_1 \leq \sigma_2$.
(In the lecture, we derived this result only for finite games.)

Exercise 1.5. A finite two-player game $G = (\{1, 2\}, \mathbf{S}, \mathbf{u})$ with utility matrices A and B is called symmetric if $A = B^T$. A mixed strategy profile (x, y) is symmetric if $x = y$ and an equilibrium is symmetric if it is a symmetric strategy profile. Show that mixed extensions of symmetric games have a symmetric Nash equilibrium.

Exercise 1.6. Consider the mixed extension of the two-player zero-sum game “higher number wins” with $S_1 = S_2 = \mathbb{N}$. The player who chose the higher number wins and receives a utility of 1; the losing player receives a utility of -1 . In case of a draw both players get utility 0.

- (a) Show that this game does not have a Nash equilibrium.
- (b) Modify the rules of the game such that the player with the higher number wins only if the higher number is less than three times the lower number, and loses otherwise. Show that the modified game has a mixed Nash equilibrium.