ADM III: Online Optimization, WS 2013/2014
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# Problem Sheet 4

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**Exercise 1** (SRPT)**.** Recall the Algorithm *Shortest Remaining Processing Time (SRPT)* that we introduced in the lecture for online scheduling on a single machine $1|\,r_j,\text{pmtn}\,|\sum_j C_j$.

> **SRPT**: At any time $t$, schedule an available unfinished job with shortest remaining processing time.

Prove that this algorithm solves the problem $1|\,r_j,\text{pmtn}\,|\sum_j C_j$ optimally.

**Exercise 2** (WSRPT)**.** Consider the weighted version of the above single-machine scheduling problem, $1|\,r_j,\text{pmtn}\,|\sum_j w_j C_j$. A natural generalization of the online algorithm SRPT is as follows.

> **WSRPT**: At any time $t$, schedule an available unfinished job with maximum ratio of weight over remaining processing time, $\frac{w_j}{p_j(t)}$.

(a) Prove an upper bound of 2 or less on the competitive ratio of WSRPT.

(b) What is the largest lower bound on the competitive ratio of WSRPT that you can prove?

**Exercise 3** (Resource augmentation)**.** In the lecture we considered the following semi-online scheduling problem: There are given $m$ identical parallel machines. Jobs with processing times and deadlines arrive at their release dates online over time, and we we may assume that there exists a feasible offline schedule for this instance on $m$ unit-speed machines. The task is to design an online algorithm that is guaranteed to find a feasible schedule for any such instance.

In the lecture we have seen that no online algorithm can guarantee to find a feasible solution without any resource augmentation, i.e., extra speed or extra machines.

(a) Prove a lower bound on the number of extra machines (of unit speed) that any online algorithm needs.

*Hint:* Try to adopt the idea for the general lower on the extra speed for a fixed number of $m$ machines.

(b) Prove the following theorem.

**Theorem.** EDF is no speed-$s$ algorithm for any $s < 2 - \frac{1}{m}$.

**Exercise 4** (LP formulation for the ski rental problem)**.** Recall the ski rental problem. A skier has to decide whether to rent or buy skis not knowing how many days she will be able to ski. Renting skis costs 1 Euro per day, while buying skis costs $B$ Euros.

Formulate the offline version of this problem as a linear program. (The offline ski rental problem is so simple that it seems a bit unnatural to do that. However, this formulation will be very useful in the next lecture.) Relax the integrality constraint and give the dual program.