ADM III: Online Optimierung, WS 2013/2014
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Problem Sheet 3

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The work function algorithm WFA for a metrical task system $(\mathcal{M} = (S, \tau), \mathcal{R})$ selects a state s_i in each step, for which

$$\begin{aligned} s_i &\in \arg\min_{x \in S}\{w_i(x) + \tau(s_{i-1}, x)\}, \\ w_i(s_i) &= w_{i-1}(s_i) + r_i(s_i). \end{aligned}$$

Recall that

$$\begin{aligned} w_0(s) &= \tau(s_0, s) \\ w_i(s) &= \min_{x \in S}\{w_{i-1}(x) + r_i(x) + \tau(x, s)\}. \end{aligned}$$

Using $\text{WFA}_i \leq B_i - B_{i-1}$, we were able to show

$$\begin{aligned} \text{WFA} &= \sum_{i=0}^{n} \text{WFA}_i \\ &\leq B_n - B_0 \\ &\leq (2N-1) \max_{s \in S} w_n(s) - B_0 \\ &= (2N-1) \max_{s \in S} \min_{x \in S} (w_{n-1}(x) + r_n(x) + \tau(x, s)) - B_0 \\ &\leq (2N-1) \min_{x \in S} (w_{n-1}(x) + r_n(x)) + \max_{x,y \in S} \tau(x, y) - B_0 \\ &\leq (2N-1) \min_{x,s \in S} (w_{n-1}(x) + r_n(x) + \tau(x, s)) + \alpha(\mathcal{M}) \\ &= (2N-1) \min_{s \in S} w_n(s) + \alpha(\mathcal{M}) \\ &= (2N-1)\text{OPT} + \alpha(\mathcal{M}) \end{aligned}$$

i.e., WFA is $(2N-1)$-competitive.

Exercise 1. Show that $\text{WFA}_i \leq B_i - B_{i-1}$.

Exercise 2. Show that the k-server problem is a special case of a metrical task system. What is the competitive ratio we would get by applying general MTS algorithms?