ADM III: Online Optimization, WS 2013/2014
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Problem Sheet 2

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Exercise 1. Show that the paging algorithms LIFO and LFU have an unbounded competitive ratio.

Exercise 2. Prove that FIFO is a k-competitive algorithm for online paging.

Exercise 3 (The cow path problem)**.** A cow is standing at a river with green yummy grassland at the other side. The cow is desperately looking for the bridge across the river. Unfortunately, it does not know which way to go: left or right? What would be the best strategy for the hungry cow in order to find the way across the river (via the bridge) as fast as possible?

This problem can be modeled as the search for an unknown point $x \in \mathbb{R}$ starting from the origin 0. The optimal offline strategy knows the position of the bridge and, thus, the value of the optimal solution (move straight from the origin to x) is simply the distance $|x|$.

Assume that $|x| \geq 1$. Try to construct an 9-competitive algorithm for the cow path problem.

Hint: Consider an algorithm that moves first $\alpha > 1$ units of length to the right, then goes back to the origin, from where it heads α^2 units of length to the left, then back to the origin, and α^3 to the right, etc. The ith turning point of that algorithm is $(-1)^{i+1}\alpha^i$. What is the competitiveness of that algorithm? How would you choose α?

Exercise 4. Consider the k-th harmonic number $H_k := \sum_{i=1}^{k} \frac{1}{i}$. Show that $\ln k < H_k \leq 1 + \ln k$.