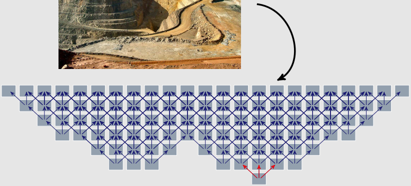
Outline

Outline

Open Pit Mining Production Scheduling

Input



- ▷ block model with precedences
- ▷ deterministic block content
- ▷ ore prices and production costs
- ▷ equipment capacities

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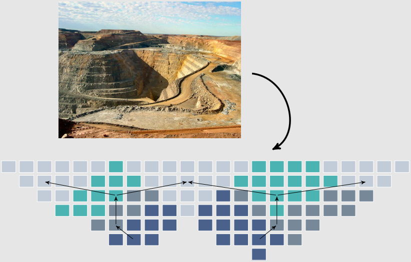

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Open Pit Mining Production Scheduling

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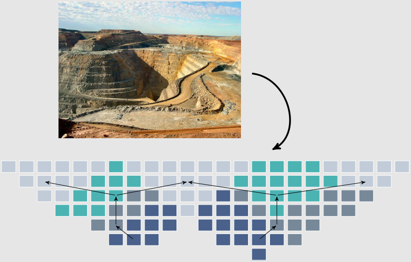

- ▷ block model with precedences
- ▷ deterministic block content
- ▷ ore prices and production costs
- ▷ equipment capacities
- ▷ possibly aggregation of blocks

Problem

Find an excavation schedule with

- ▷ max. net present value

satisfying

- ▷ precedence constraints,
- ▷ resource constraints.

Outline

Open Pit Mining Production Scheduling

- Problem description
- Ultimate pit limits
- Time-indexed IP formulations

Precedence Constrained Knapsack Substructures

- Precedence Constrained Knapsack Problem
- PCKs in open pit mining
- Valid inequalities for the Precedence Constrained Knapsack Polytope

Getting started: computing ultimate pit limits

Given: Profit/negative cost w_i of mining each block $i \in \mathcal{N} = \{1, \dots, N\}$.

Seek: Precedence-feasible set of blocks $X \subseteq \mathcal{N}$ with max. $\sum_{i \in X} w_i$.

(If $i \in X$ than all predecessors of i are in X.)

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Observation: This is a max-weight closure problem in the precedence graph

$$\big(\mathcal{N}, \mathcal{S} = \{(i,p) \in \mathcal{N} \times \mathcal{N} \mid p \text{ predecessor of } i\}\big)$$

with node weights w_i.

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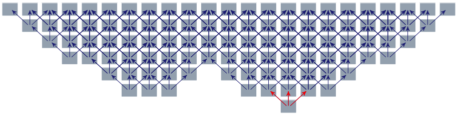
- Solve e.g. by min-cut algorithm in a slightly extended graph:



Outline

A simple time-indexed IP formulation

$$\max \quad NetPresentValue(x)$$

$$\begin{aligned}
\text{s.t.} \qquad x_{i,t-1} &\le x_{i,t} && \text{for } i \in \mathcal{N},\ t = 1, \dots, T-1 \\
x_{i,t} &\le x_{k,t} && \text{for } i \in \mathcal{N}, k \in \mathcal{P}(i), t = 0, \dots, T-1 \\
\sum_i R_i x_{i,t} &\le (t+1)M && \text{for } t = 0, \dots, T-1 \\
\sum_i O_i x_{i,t} &\le (t+1)P && \text{for } t = 0, \dots, T-1 \\
x_{i,t} &\in \{0,1\} && \text{for } i \in \mathcal{N},\ t = 0, \dots, T-1 \\
x_{i,-1} &= 0 && \text{for } i \in \mathcal{N}
\end{aligned}$$

Outline

Precedence Constrained Knapsack Problem

Given:

- Items $i \in \mathcal{N}$ with sizes a_i and values c_i
- Knapsack capacity b
- Precedence order $\mathcal{S} \subseteq \mathcal{N} \times \mathcal{N}$ (transitively reduced)

Feasible Solution:

- set $X \subseteq \mathcal{N}$ with $\sum_{i \in X} a_i \leqslant b$ and
- X is closed under $\mathcal{S}$ (If $i \in X$ and $(i, j) \in \mathcal{S}$, then $j \in X$.)

Goal: maximize $\sum_{i \in X} c_i$

Important substructure in many applications:

- Resource constrained scheduling
- Multi-level facility location, access & distribution network design
- Routing problems: confluent flows, Internet routing, ...
- ...

Outline

PCKs in open pit mining

$$\max \quad NetPresentValue(x)$$

$$
\begin{array}{lll}
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PCKs in open pit mining

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\end{aligned}
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$$
x_{i,t} \in \{0,1\} \qquad \text{for } i \in \mathcal{N}
$$

- For each time period t, the OPMPSP formulation contains two precedence constrained knapsacks (for mining and processing constraints).

Outline

Precedence Constrained Knapsack Polytope

$$PCKP := \text{conv}\Big\{x \in \{0,1\}^{\mathcal{N}} \mid \sum_{i \in \mathcal{N}} a_i x_i \leqslant b, \quad \text{(i)}$$
$$x_i \leqslant x_j \;\; \text{for } (i,j) \in \mathcal{S} \Big\} \quad \text{(ii)}$$

Polyhedral structure of $PCKP$ well studied:

- Dimension, Conditions for (i) and (ii) to be facet-defining
- Knapsack-based inequalities [Boyd'93, Park-Park'97]
  - Induced cover inequalities
  - (Induced) k-cover inequalities
  - (Induced) (1,k)-configuration inequalities
- Sequential lifting results [v.d.Leensel-v.Hoesel-v.d.Klundert'99]

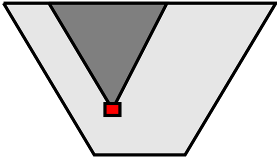
But:

- Nothing implemented in MIP solvers

Variable fixing

Notation

- $Pred(i) := \{j : j \text{ predecessor of } i\} \cup \{i\}$
  (Items that must be included with i.)
- $A(i) := \sum_{j \in Pred(i)} a_j$
  (Total size induced by i.)



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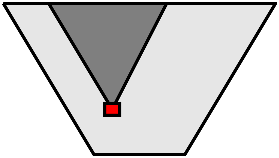

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Observation: If $A(i) > b$, then $x_i = 0$ for all $x \in PCKP$.

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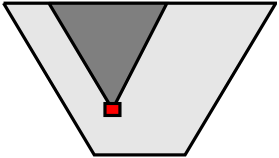

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Variable elimination scheme

- If $A(i) > b$, fix $x_i = 0$.

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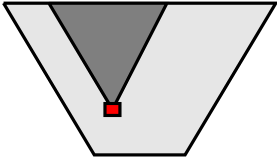

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Variable elimination scheme

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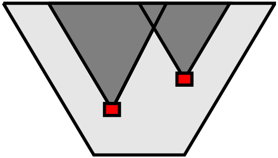
Remarks

- Can compute *all* $A(i)$ in $\mathcal{O}(\mathcal{S})$.
- Fixing is very efficient in practice (reduces LP gaps substantially).
- CPLEX does *not* find these fixings (even with aggressive probing).

Pairwise conflicts

Apply same idea to pairs of items:

- $A(i,j) := \sum_{k \in Pred(i) \cup Pred(j)} a_k$
  (Total size of i, j and predecessors.)



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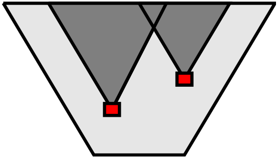

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Observation: If $A(i,j) > b$, then $x_i + x_j \leqslant 1$ for all $x \in PCKP$.

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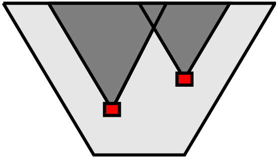

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Observation: If $A(i,j) > b$, then $x_i + x_j \leqslant 1$ for all $x \in PCKP$.

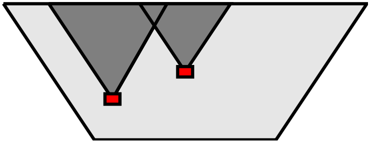
- ▷ $\{i,j\}$ with $A(i,j) > b$: Induced cover of size 2
- ▷ Induced cover: $X \subset \mathcal{N}$ with $A(X) > b$
- ▷ [Boyd'93, Park-Park'97] Conditions when induced cover inequalities are facet-defining (for low-dimensional faces of PCKP)
- ▷ [v.d.Leensel-v.Hoesel-v.d.Klundert'99] Lifting facet-defining inequalities for low-dimensional faces of PCKP to facets of PCKP

Clique inequalities

Idea

- Consider clique graph $CG = (\mathcal{N}, E)$ with $E := \{ij \mid A(i,j) > b\}$

Precedence order



Clique graph CG

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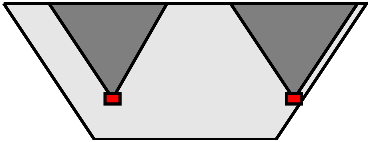

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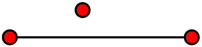

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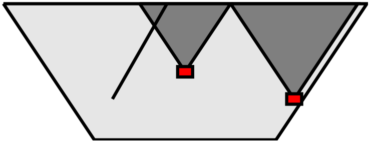

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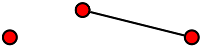

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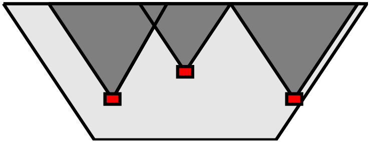

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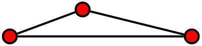

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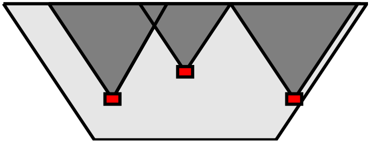

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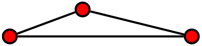

Observation: Any valid inequality for $STAB(CG)$ is valid for $PCKP$.

Clique inequalities

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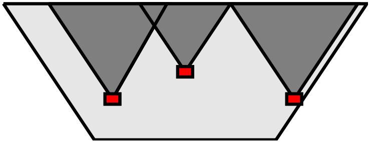

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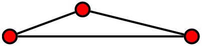


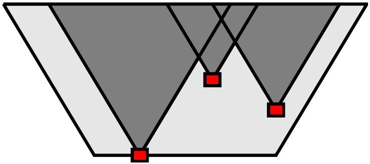
Observation: Any valid inequality for $STAB(CG)$ is valid for $PCKP$.

Corollary: Let C be a clique in CG. Then (1) is valid for $PCKP$.

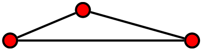
$$\sum_{i \in C} x_i \leqslant 1 \qquad (1)$$

Cliques with common predecessors

Precedence order



Extended clique graph
CG'



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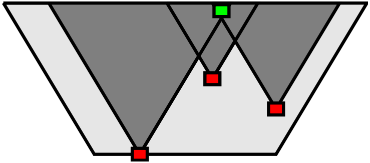

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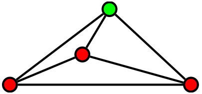

Observation: Let C be a clique in CG with $P(C) = \bigcap_{i \in C} Pred(i) \neq \emptyset$. Then, for any $i \in P(C)$,

$$\sum_{j \in C} x_j \leqslant x_i \tag{2}$$

is valid for $PCKP$.

Cliques with common predecessors

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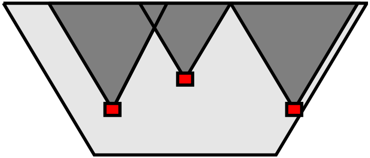

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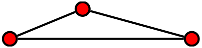

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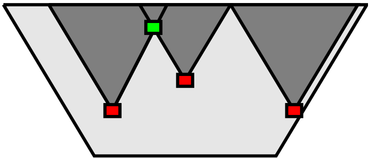

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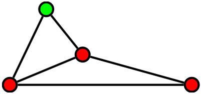

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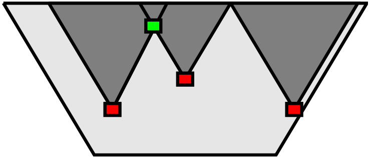

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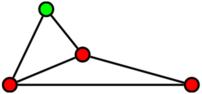

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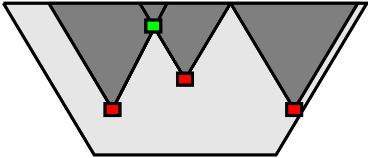
$$\sum_{j \in C} x_j + (1 - x_i) \leqslant 1 \tag{2}$$

is valid for $PCKP$.

Cliques with common predecessors

Precedence order



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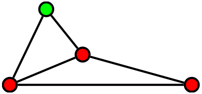

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Separation of clique-based inequalities

Given: $x \in [0,1]^{\mathcal{N}}$ fractional

Seek:

(1) clique C in CG such that $\sum_{j \in C} x_j > 1$, or

(2) clique C in CG and $i \in P(C)$ such that $\sum_{j \in C} x_j > x_i$

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- Separation of (1) and (2) is equivalent to finding a *maximum-weight clique* in CG' with node weights $w_k := x_k$ and $w_{k'} := 1 - x_k$ for all $k \in \mathcal{N}$
  - $w(C^*) \leqslant 1$: all inequalities (1) and (2) satisfied
  - $w(C^*) > 1$ and $C^* \subseteq \mathcal{N}$: inequality (1) violated
  - $w(C^*) > 1$ and $C^* = C \cup \{\bar{i}\}$: inequality (2) violated

Separation of clique-based inequalities

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  - $w(C^*) > 1$ and $C^* \subseteq \mathcal{N}$: inequality (1) violated
  - $w(C^*) > 1$ and $C^* = C \cup \{\bar{i}\}$: inequality (2) violated
- NP-hard, but: Efficient MWC implementations exist in any state-of-the-art MIP solver.
- Ideally: extend the clique graph of your MIP solver.

More details on facets and computational results: see e.g. Boland-Froyland-Fricke-Sotirov'06, Bley-Boland-Fricke-Froyland'09.

Summary

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