Theorem 11.37: If P is a rational polyhedron, then $P' := \{x \in P \mid x$ satisfies every Gomory-Chvátal cut for $P\}$ is also a rational polyhedron.

Proof: Let $P = \{x \mid Ax \leq b\}$ with A and b integral. Then P' is defined by $Ax \leq b$ and the set $S := \{ (y^T A) x \leq \lfloor y^T b \rfloor \mid \vec{0} \leq y < \vec{1}$ and $y^T A$ integral$\}$.
To prove this desired result, let $w^T x \leq \lfloor t \rfloor$ be any Gomory-Chvátal cut, derived from $Ax \leq b$ with the nonnegative vector $\bar{y}$.
We will show that $w^T x \leq \lfloor t \rfloor$ can be written as a linear combination of inequalities from $Ax \leq b$ plus an inequality in S.
($\Rightarrow$ any inequality not in S is redundant in the definition of P'.)
Let $\bar{y}' = \bar{y} - \lfloor \bar{y} \rfloor$. Then $w' = (\bar{y}')^T A = \underbrace{w}_{=\bar{y}^T A} - (\lfloor \bar{y} \rfloor)^T A$ is integral
and $t' = (\bar{y}')^T b = t - (\lfloor \bar{y} \rfloor)^T b$
differs from t by an integral amount.
Thus, the cut $(w')^T x \leq \lfloor t' \rfloor$ derived with $\bar{y}'$, together with $(\lfloor \bar{y} \rfloor)^T A \; x \leq (\lfloor \bar{y} \rfloor)^T b$ (which is a nonnegative combination of $Ax \leq b$) sum to the cut $w^T x \leq t$.

$\square$

Enumeration:

A TSP-instance on n points has $\frac{(n-1)!}{2}$ different tours.

Thus, for $\begin{cases} n = 15, & \text{we have more than 43 milliards tours.} \\ n = 18, & \text{— " — 177 billions tours.} \end{cases}$

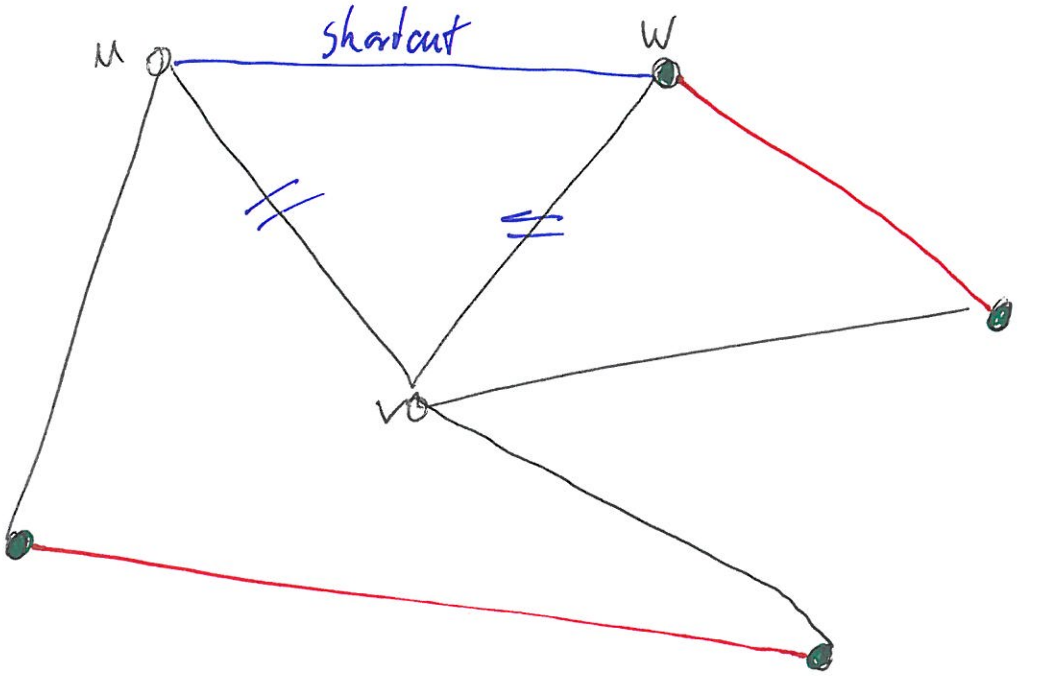
If you (or your computer) could ~~compute~~ enumerate 30 points in one hour, you would need for two additional points almost 1000 hours ($\approx$ 40 days)

In practice / records:

- In 2006, Cook was able to compute an optimal tour on 85.900 points
- It is possible to compute tours on millions of points that are of length at most 1% away from the optimum.
- TSPLIB library provides test instances.

Tree $T=(V,E)$

Matching M



Note: $|W|$ is even.

Note: In each iteration, $H=(V,J)$ has an Euler tour;
choose $\{u,v\}$ and $\{v,w\}$ as consecutive edges on such a tour;
$\rightarrow$ "Shortcut"