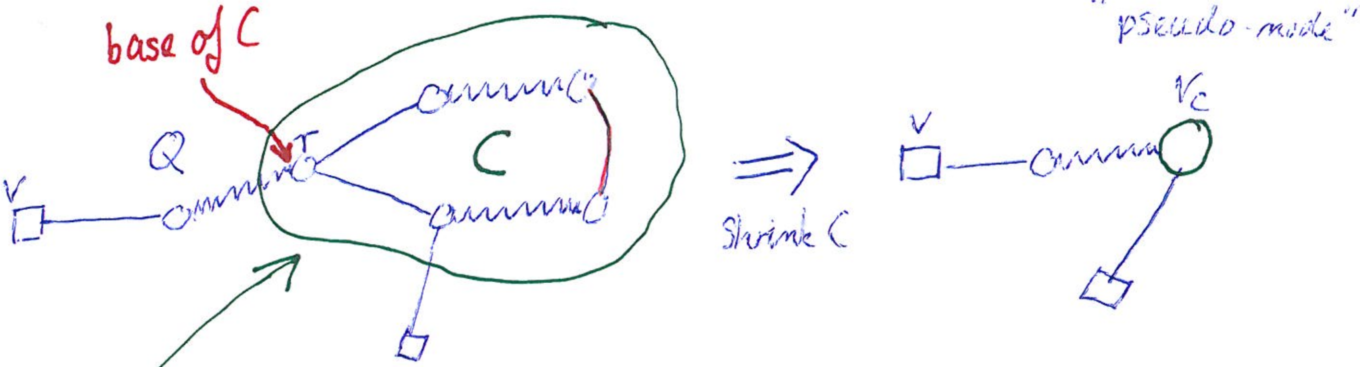
Problem: Odd M-alternating circuits!

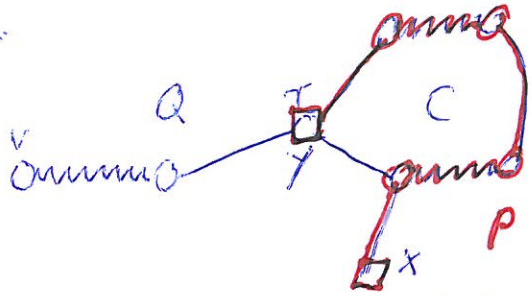


M-blossom: M-alternating odd circuit

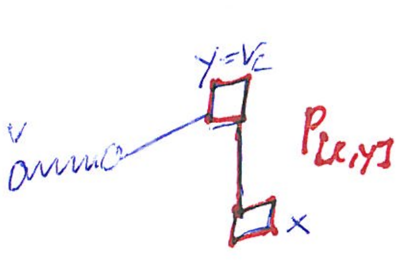
Lemma 13.13: M max. in $G \Leftrightarrow M'$ max. in G'

Proof: „$\Rightarrow$" Every matching N' in G' corresponds to a matching N in G with $|N| = |N'| + \frac{|V(C)|-1}{2}$.
(N is obtained from N' by expanding (blowing up) C and add matching edges accordingly.)

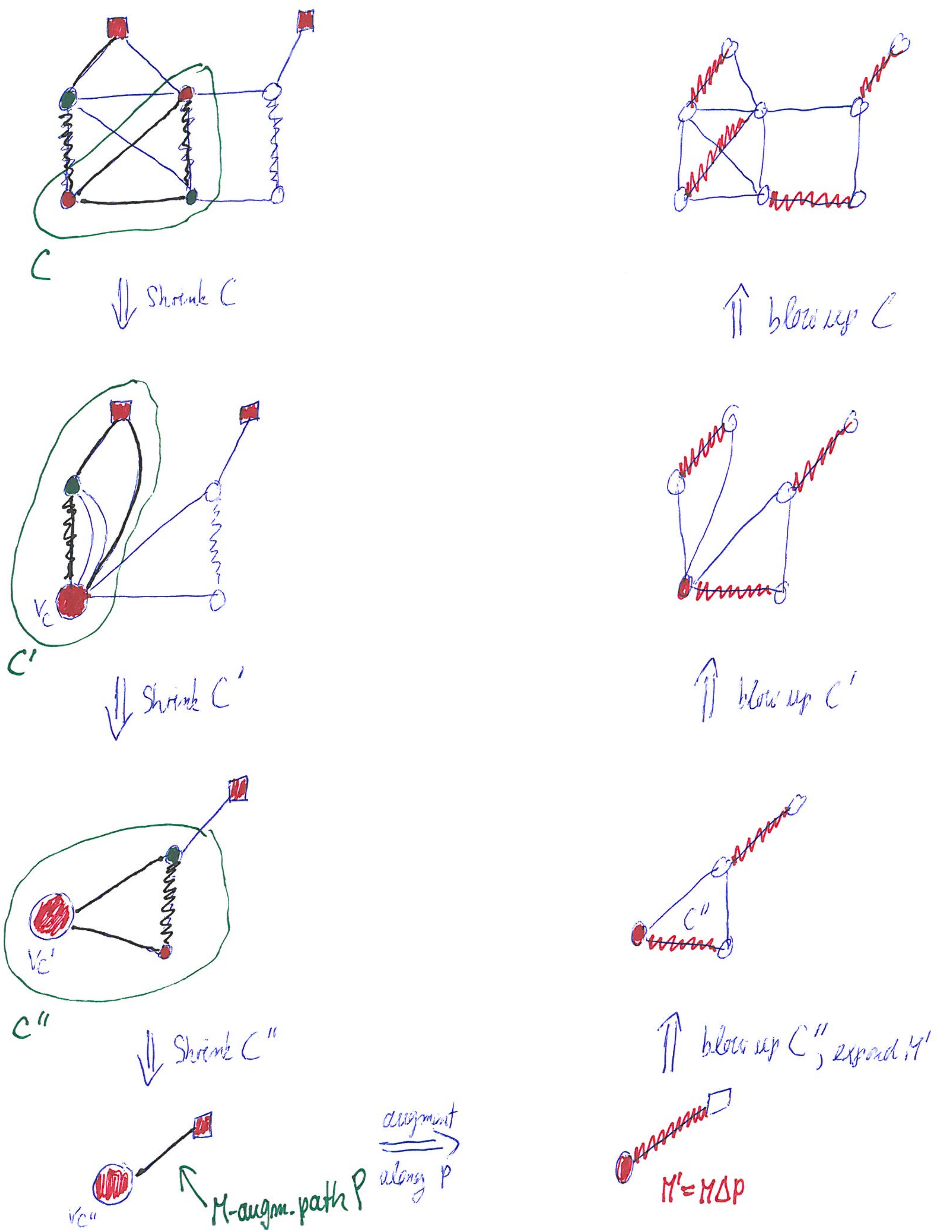
„$\Leftarrow$" Assume that M is not maximum.
Then also $\underbrace{M \Delta Q}_{=:N}$ is not maximum.
$\underset{\text{(Berge)}}{\Rightarrow}$ $\exists$ an N-augmenting path P in G.



One endpoint x of P does not belong to C. If P and C are disjoint, let y be the other endpoint of P. Otherwise, let y be the base of M-blossom C.
Consider matching N' in G' corresponding to N.
$\Rightarrow P_{[x,y]}$ is N'-augm. path in G'. ↯



Matching Algorithm (Example):



Theorem 13.15: Edmonds' matching algorithm works correctly.

Proof: If the algorithm stops in step 2, we have to argue that the current matching is optimal.
Let X be the set of inner nodes of F.
Then $G-X$ consists of isolated nodes.
Thus, $q_G(X) = |V(G-X)|$
By Obs. 13.10, any matching in G leaves at least $|V(G-X)|-|X|$ nodes uncovered.
On the other hand, $|V(G-X)|-|X|$ is the number of trees and thus the number of M-exposed nodes $\Rightarrow$ M is optimal matching in current graph.
Use Lemma 13.13 if shrinking occured before step 2.
In this case, blow blossoms in reverse order and expand matching accordingly. $\square$