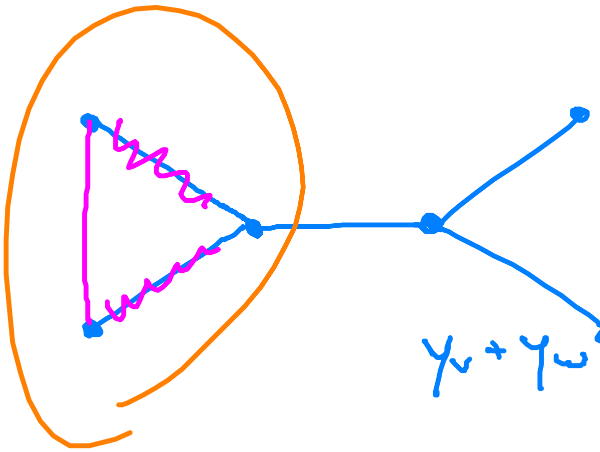

$$y_v + y_w + \sum_{e \in D} y_D \le c_e$$

## Proof of Theorem 13.32:

Let $\tilde{\mathcal{T}}^k$ and $\varepsilon^k$ be $\tilde{\mathcal{T}}$ and $\varepsilon$ in iteration $k$ of the algorithm. Consider $e \in E(F^*)$, $e = \{v,w\}$ and let $v \in T_i \in \tilde{\mathcal{T}}^k$, $w \in T_j \in \tilde{\mathcal{T}}^k$. Set

$$c_e^k := \begin{cases} 0 & \text{if } T_i = T_j \\ \varepsilon^k \cdot (\operatorname{parity}(T_i) + \operatorname{parity}(T_j)) & \text{otherwise} \end{cases}$$

(Increase of left-hand side of dual constraint corresp. to edge $e$)

Then
$$\sum_{k=1}^{t} c_e^k = c_e$$

**Claim:** $\displaystyle\sum_{e \in E(F^*)} c_e^k \le 2 \cdot \varepsilon^k \cdot |\{T \in \mathcal{T}^k \mid |T| \text{ odd}\}| \quad \forall k$

$$\left( \Longrightarrow \sum_{e \in E(F^*)} c_e \le 2 \cdot \left( \sum_{v \in V} y_v + \sum_{D} y_D \right) \square \right)$$

## Proof of Claim:

Let $\bar{F}^*$ be obtained from $F^*$ by shrinking each set $T \in \tilde{\mathcal{L}}^k$ to a single node.
Notice that $\bar{F}^*$ is a forest.
We label the nodes of $\bar{F}^*$ as even/odd depending on the parity of the corresp. node set $T \in \tilde{\mathcal{L}}^k$:

$$V(\bar{F}^*) = V^{odd} \,\dot\cup\, V^{even}$$

Since $\bar{F}^*$ does not contain even edges, no leaf of $\bar{F}^*$ is in $V^{even}$.
We prove that $\boxed{\sum_{v \in V^{odd}} \deg(v) \le 2|V^{odd}|}$ (*)

Notice that

$$\sum_{v \in V(\bar{F}^*)} \deg(v) = 2 \cdot |E(\bar{F}^*)| \le 2|V(\bar{F}^*)|$$
$$= 2|V^{odd}| + 2|V^{even}|.$$

Moreover:

$$\sum_{v \in V^{odd}} \deg(v) = \sum_{v \in V(\bar{F}^*)} \deg(v) - \sum_{v \in V^{even}} \deg(v)$$

$$\le 2|V^{odd}| + 2|V^{even}| - 2|V^{even}|$$
$$= 2|V^{odd}| \qquad \boxed{(*)}$$

Since $\sum_{e \in E(F^*)} c_e^k = \varepsilon^k \cdot \sum_{v \in V^{odd}} \deg_{\bar{F}^*}(v)$

$\le 2\varepsilon^k \cdot |V^{odd}|$

the claim follows. $\square$

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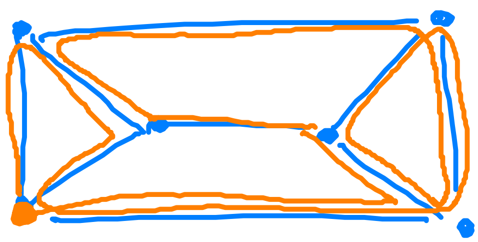
Example of Postman Problem:



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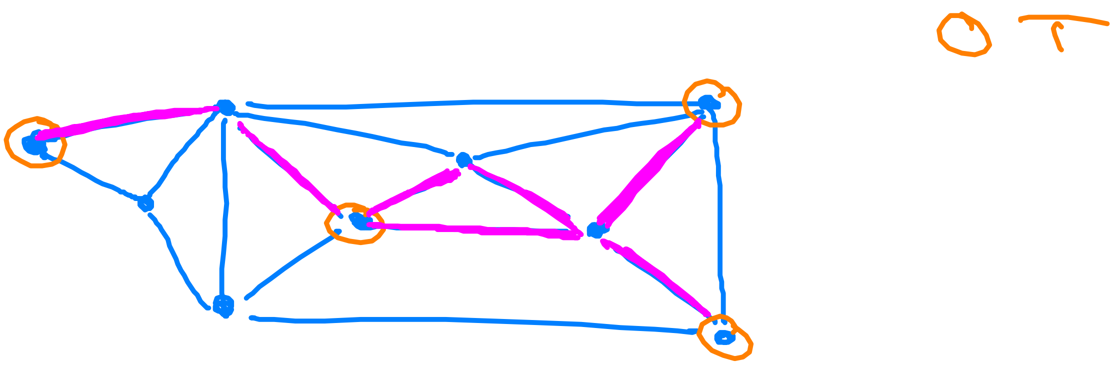
Example: T-join

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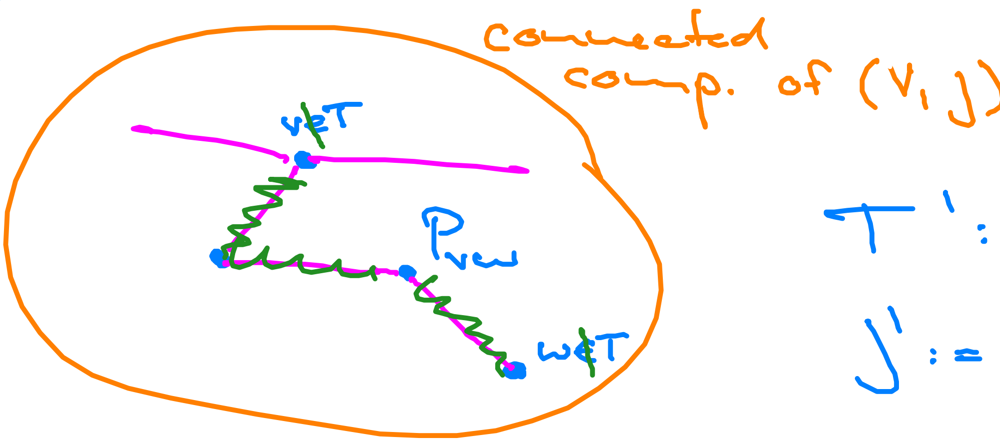
## Sketch of proof of Lemma 13.37:




$T' := T \setminus \{v, w\}$

$J' := J \setminus E(P_{vw})$

$J'$ is $T'$-join

Dealing with negative edge weights:


$T' := T \triangle \{v, w\}$

replace $c_e$ by $-c_e$