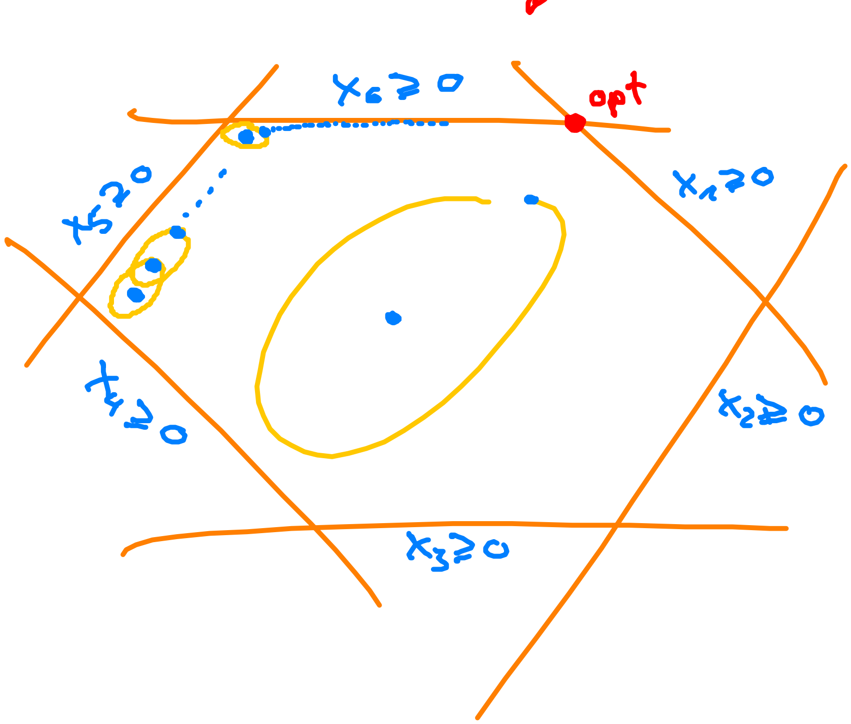
Affine Scaling Alg.



$A \cdot x = b$

Proof of Theorem 12.6:

$$G(x,s) = q \cdot \log(s^T \cdot x) - \sum_{j=1}^{n} \log x_j - \sum_{j=1}^{n} \log s_j$$

$$= n \cdot \log(s^T x) - \sum_{j=1}^{n} \log x_j - \sum_{j=1}^{n} \log s_j + (q-n)\log(s^T x)$$

$$= \underbrace{n \cdot \log(s^T x) - \sum_{j=1}^{n} \log(x_j \cdot s_j)}_{\substack{\text{minimum is attained} \\ \text{for } x_j \cdot s_j = \frac{s^T x}{n} \ \forall j}} + (q-n) \cdot \log(s^T x)$$

$$\geq n \cdot \log n + (q-n) \cdot \log(s^T x)$$

$$\Longrightarrow n \cdot \log(s^T x) - \sum_{j=1}^{n} \log x_j - \sum_{j=1}^{n} \log s_j \geq n \log n \quad (*)$$

Suppose that $G(x^{k+1}, s^{k+1}) - G(x^k, s^k) \leq -\delta \ \forall k$

$$\Longrightarrow G(x^K, s^K) - G(x^0, s^0) \leq -K \cdot \delta.$$

Our choice of K yields

$$G(x^K, s^K) \leq -(q-n) \cdot \log 1/\varepsilon + n \log n$$

Using the def. of $G(x,s)$ and (*) yields:

$$G(x^k, s^k) \geq n \log n + (q-n) \cdot \log (s^{k^T} \cdot x^k)$$

$$\Rightarrow \quad (s^k)^T \cdot x^k \leq \varepsilon$$

$\square$