

# Parametric Programming:

Example:

Set up simplex tableau:

	$\theta$	$-3+2\theta$	$3-\theta$	1	0	0
$x_4$	5	1	2	-3	1	0
$x_5$	7	2	1	-4	0	1

If  $-3+2\theta \geq 0$  and  $3-\theta \geq 0$ , then solution is optimal  $\Rightarrow g(\theta) = 0$  for  $\frac{3}{2} \leq \theta \leq 3$

Consider  $\theta > 3 \Rightarrow \bar{C}_2 < 0$ , do pivoting step:

	$-7.5+2.5\theta$	$-\frac{3}{2}+\frac{5}{2}\theta$	0	$\frac{1}{2}-\frac{3}{2}\theta$	$-\frac{3}{2}+\frac{1}{2}\theta$	0
$x_2 =$	$\frac{5}{2}$	$\frac{1}{2}$	1	$-\frac{3}{2}$	$\frac{1}{2}$	0
$x_5 =$	$\frac{9}{2}$	$\frac{3}{2}$	0	$-\frac{5}{2}$	$-\frac{1}{2}$	1

optimal for  $3 \leq \theta \leq \frac{11}{3}$

$$g(\theta) = 7.5 - 2.5\theta \quad \text{for } 3 \leq \theta \leq \frac{11}{3}$$

If  $\theta > \frac{11}{3}$ , then  $\bar{c}_3 < 0$  and the problem is unbounded (we have found an extreme ray, i.e. 3rd basic direction, with negative cost).

Consider now the situation for  $\theta < \frac{11}{2}$  (go back to initial tableau):

$$\bar{c}_1 < 0$$

		0	$4.5 - 2\theta$	$-5 + 4\theta$	0	$1.5 - \theta$
$x_2$	$\frac{3}{2}$	0	$\frac{3}{2}$	-1	1	$-\frac{1}{2}$
$x_1$	$\frac{7}{2}$	1	$\frac{1}{2}$	-2	0	$\frac{1}{2}$

optimal for  $\frac{5}{4} \leq \theta \leq \frac{11}{2}$

$$g(\theta) = 10.5 - 7 \cdot \theta \quad \text{for } \frac{5}{4} \leq \theta \leq \frac{11}{2}$$

For  $\theta < \frac{5}{4}$ ,  $\bar{c}_3 < 0$  and the problem is unbounded.

$$g(\theta) = -\infty \quad \text{for } \theta < \frac{5}{4}$$

This approach works for arbitrary parametric programs....

If an anti-cycling rule is being applied (e.g. lexicographic rule), then the algorithm will terminate after finitely many steps since it visits every basic feasible solution at most once...

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