

$$y_v + y_w + \sum_{D} y_D \leq c_e$$

Proof of Theorem 13.32:

Let \tilde{T}^k and ε^k be \tilde{T} and ε in iteration k of the algorithm. Consider $e \in E(F^*)$, $e = \{v, w\}$ and let $v \in T_i \in \tilde{T}^k$, $w \in T_j \in \tilde{T}^k$. Set

$$c_e^k := \begin{cases} 0 & \text{if } T_i = T_j \\ \varepsilon^k \cdot (\text{parity}(T_i) + \text{parity}(T_j)) & \text{otherwise} \end{cases}$$

(Increase of left-hand side of dual constraint corresp. to edge e)

Then

$$\sum_{k=1}^t c_e^k = c_e$$

Claim: $\sum_{e \in E(F^*)} c_e^k \leq 2 \cdot \varepsilon^k \cdot |\{T \in \tilde{T}^k \mid |T| \text{ odd}\}| \quad \forall k$

$$\left(\Rightarrow \sum_{e \in E(F^*)} c_e \leq 2 \cdot \left(\sum_{v \in V} y_v + \sum_D y_D \right) \square \right)$$

Proof of Claim:

Let \bar{F}^* be obtained from F^* by shrinking each set $T \in \tilde{\mathcal{T}}^k$ to a single node.

Notice that \bar{F}^* is a forest.

We label the nodes of \bar{F}^* as even/odd depending on the parity of the corresp. node set $T \in \tilde{\mathcal{T}}^k$:

$$V(\bar{F}^*) = V^{\text{odd}} \cup V^{\text{even}}$$

Since F^* does not contain even edges, no leaf of \bar{F}^* is in V^{even} .

We prove that $\boxed{\sum_{v \in V^{\text{odd}}} \deg(v) \leq 2|V^{\text{odd}}|} \quad (*)$

Notice that

$$\begin{aligned} \sum_{v \in V(\bar{F}^*)} \deg(v) &= 2 \cdot |E(\bar{F}^*)| \leq 2|V(\bar{F}^*)| \\ &= 2|V^{\text{odd}}| + 2|V^{\text{even}}|. \end{aligned}$$

Moreover:

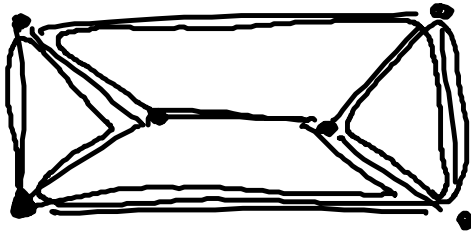
$$\begin{aligned} \sum_{v \in V^{\text{odd}}} \deg(v) &= \sum_{v \in V(\bar{F}^*)} \deg(v) - \sum_{v \in V^{\text{even}}} \deg(v) \\ &\leq 2|V^{\text{odd}}| + 2|V^{\text{even}}| - 2|V^{\text{even}}| \\ &= 2|V^{\text{odd}}| \quad \boxed{(*)} \end{aligned}$$

Since $\sum_{e \in E(F^*)} c_e^k = e^k \cdot \sum_{v \in V^{\text{odd}}} \deg_{\bar{F}^*}(v)$

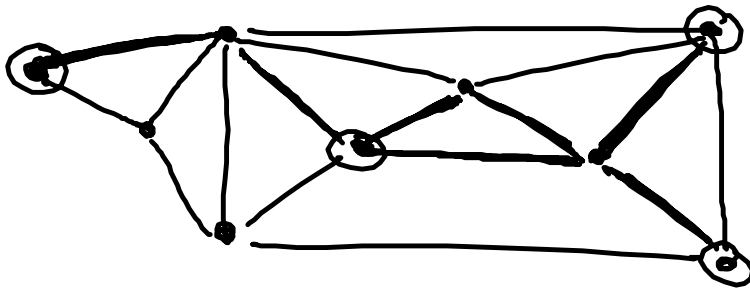
$$\leq 2\epsilon^k \cdot |V^{\text{odd}}|$$

the claim follows. \square

Example of Postman Problem:

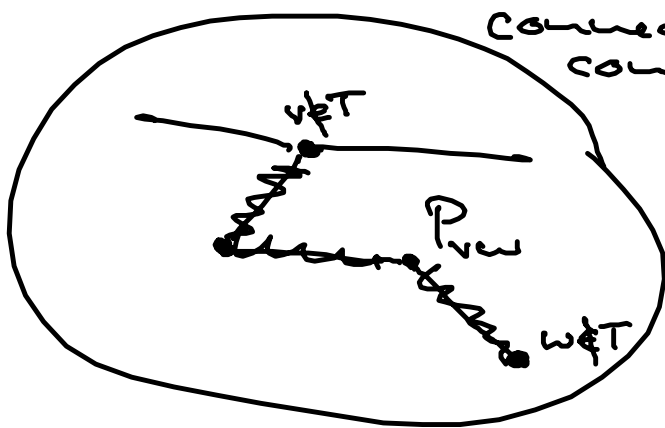


Example: T-join



O_T

Sketch of proof of Lemma B.37:



connected comp. of (V, J)

$$T' := T \setminus \{v, w\}$$

$$J' := J \setminus E(P_{vw})$$

J' is T' -join

Dealing with negative edge weights:

$v \xrightarrow{c_e < 0} w$ $T' := T \Delta \{v, w\}$
replace c_e by $-c_e$