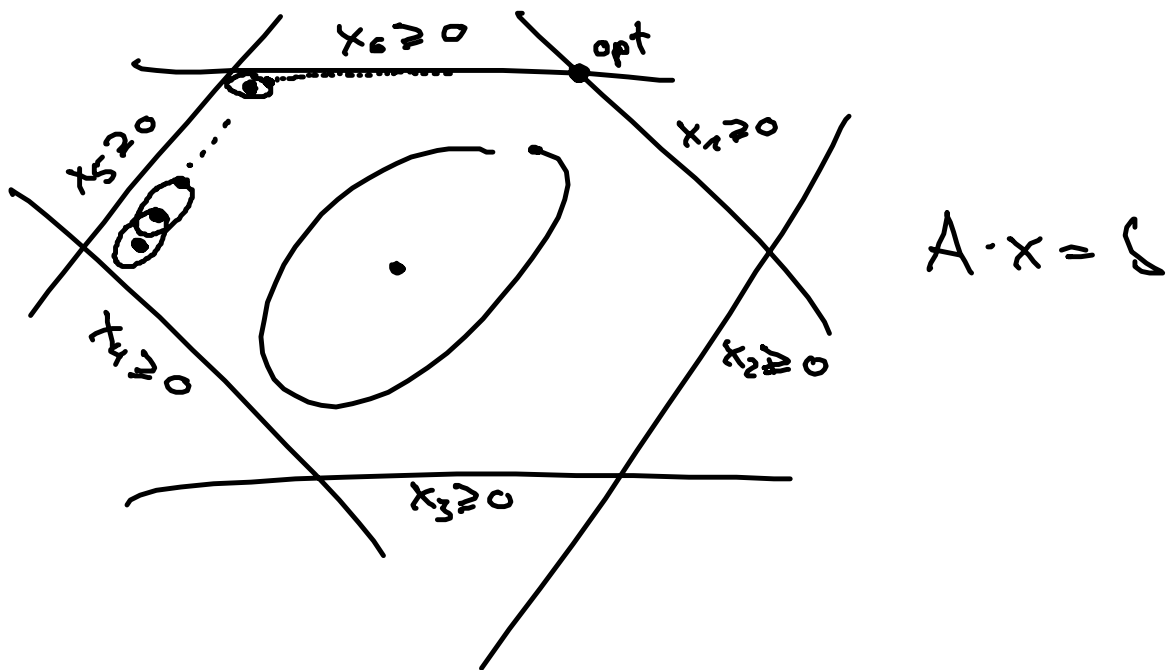


# Affine Scaling Alg.



Proof of Theorem 12.6:

$$\begin{aligned}
 G(x, s) &= q \cdot \log(s^T x) - \sum_{j=1}^n \log x_j - \sum_{j=1}^n \log s_j \\
 &= u \cdot \log(s^T x) - \sum_{j=1}^u \log x_j - \sum_{j=1}^u \log s_j + (q-u) \log(s^T x) \\
 &= \underbrace{u \cdot \log(s^T x) - \sum_{j=1}^u \log(x_j \cdot s_j)}_{\text{minimum is attained for } x_j \cdot s_j = \frac{s^T x}{u} \quad \forall j} + (q-u) \cdot \log(s^T x) \\
 &\geq u \cdot \log u + (q-u) \cdot \log(s^T x)
 \end{aligned}$$

$$\Rightarrow u \cdot \log(s^T x) - \sum_{j=1}^u \log x_j - \sum_{j=1}^u \log s_j \geq u \log u \quad (*)$$

Suppose that  $G(x^{k+1}, s^{k+1}) - G(x^k, s^k) \leq -\delta \quad \forall k$

$$\Rightarrow G(x^k, s^k) - G(x^0, s^0) \leq -K \cdot \delta.$$

Our choice of  $K$  yields

$$G(x^k, s^k) \leq -(q-u) \cdot \log \frac{1}{\epsilon} + u \log u$$

Using the def. of  $G(x, s)$  and (\*) yields:

$$G(x^k, s^k) \geq u \log u + (q-u) \cdot \log (s^k)^T \cdot x^k$$

$$\Rightarrow (s^k)^T \cdot x^k \leq \epsilon$$

□