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Discrete Optimization (ADM II)
SoSe 2012

Eigth Set of Exercises

(due date: June 14, **before** the exercise session)

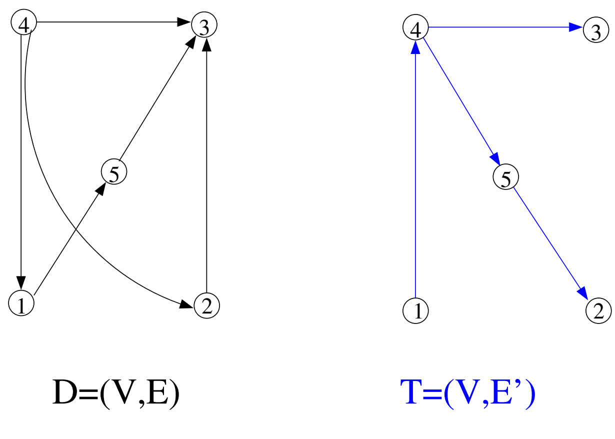
Exercise 46 (5 points)**.** Give an example of a matrix A and a vector b such that $\{x \mid Ax \leq b,\ x \geq 0\}$ is an integral polytope but A is not TUM.

Exercise 47 (5 points)**.** Use total unimodularity to prove the MAX-FLOW-MIN-CUT Theorem.

Exercise 48 (5 points)**.** Let $G = (V, E)$ be an undirected graph and $A \in \{0, 1\}^{V \times E}$ its vertex-edge incidence matrix. Show that A is TUM if and only if G is bipartite.

Exercise 49 (5 points)**.** A matrix A with entries in $\{0, 1\}$ is said to have the *consecutive-ones-property* if the nonzero entries in each column occur consecutively. Show that such a matrix is TUM.

Exercise 50 (tutorial session)**.** Consider the digraph $D = (V, E)$ and the spanning tree $T = (V, E')$ in Figure 50.



a) Write down the vertex-edge incidence matrix $A \in \{-1, 0, 1\}^{V \times E}$ of D.

b) Write down the network matrix w.r.t. D and T.

c) Draw a tree $\tilde{T}$ so that A is the network matrix w.r.t. D and $\tilde{T}$.

Exercise 51 (tutorial session)**.** Let $A \in \mathbb{R}^{m\times n}$ be TUM.

a) Convince yourself that also $-A$, $[A\ -A], [A\ I_m], A^T$, all submatrices of A, and all matrices obtained by a sequence of column- and/or row-permutations on A are TUM. (As usual, I_m is the m by m identity matrix.)

b) Given $b \in \mathbb{Z}^m$ and $l, u \in \mathbb{Z}^n$, show that the polyhedra

   - $\{x \in \mathbb{R}^n \mid Ax = b,\ x \geq 0\}$,
   - $\{x \in \mathbb{R}^n \mid Ax \leq b\}$,
   - $\{x \in \mathbb{R}^n \mid Ax \leq b,\ x \geq 0\}$,
   - $\{x \in \mathbb{R}^n \mid Ax \leq b,\ l \leq x \leq u\}$, and
   - $\{x \in \mathbb{R}^n \mid Ax = b,\ l \leq x \leq u\}$

   are integral.

Exercise 52 (tutorial session)**.** Let $A \in \mathbb{R}^{m\times n}$.

a) Show that A is TUM if and only if $[A\ I_m]$ is unimodular.

b) Let A have full row rank and B be a basis of A. Prove that $B^{-1}A$ is TUM.