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Discrete Optimization (ADM II)
SoSe 2012

Seventh Set of Exercises

(due date: June 7, **before** the exercise session)

Exercise 39 (2+2+2 points)**.** An *integer linear program (ILP)* is a linear program with the additional constraints that all variables should be integral. Formulate the following combinatorial optimization problems as ILPs:

1. Given a knapsack of capacity b and n items. Each item i has a certain weight w_i as well as a value v_i indicating how worthful it is to have this item packed into the knapsack. The task is to select a subset of items of maximum value so that that the total weight of the selected items does not exceed the capacity b.
2. Given a directed graph $G = (V, A)$, two vertices $s, t \in V$ and arc capacities $u : A \to \mathbb{Z}$, find an integral s,t-flow of maximum value.

What can you say about the complexity of the problems?

Exercise 40 (5 points)**.** Show that the problem to find the greatest common divisor of two integers $a, b \in \mathbb{Z}$ can be formulated as the following ILP:

$$\mathrm{ggT}(a, b) = \min\{ax + by \mid ax + by \geq 1,\ x, y \in \mathbb{Z}\}.$$

Exercise 41 (2+3 points)**.** Let $G = (V, E)$ be a bipartite graph. Use Lemma 14.9 to show that all vertices of the polytope $P := \{x \in \mathbb{R}^E_+ \mid x(\delta(v)) = 1\ \forall v \in V\}$ are integral.

Exercise 42 (4 points)**.** Let $G = (V, E)$ be a (not necessary bipartite) graph. Show that a vector $x \in \mathbb{R}^E$ is a vertex of $P := \{x \in \mathbb{R}^E_+ \mid x(\delta(v)) = 1\ \forall v \in V\}$ if and only if $x \in \{0, \frac{1}{2}, 1\}^E$ and the edge set $E^x := \{e \in E \mid x_e \notin \mathbb{Z}\}$ form vertex disjoint odd circuits.

Exercise 43 (tutorial session)**.** A coloring of the vertices of an undirected graph $G = (V, E)$ is called *feasible* if no two adjacent vertices have the same color. Formulate the problem to feasibly color all vertices in G with as few different colors as possible as ILP.

Exercise 44 (tutorial session)**.** Let P be a polyhedron.

- Show that P is pointed whenever P is bounded. (Since the arguement is used in the proof of Theorem 14.10, you should not use it here.)
- Let P be a polyhedron with $P \subseteq \{x \mid x \geq 0\}$. Show that P is pointed.

Exercise 45 (tutorial session)**.** Give an example of a non-bipartite graph $G = (V, E)$ such that $P := \{x \in \mathbb{R}^E_+ \mid x(\delta(v)) = 1\ \forall v \in V\}$ has a vertex with at least one non-integral component.