Assignment 2

Discussion: Thursday, May 16th.

Exercise 1. Let $L(\mathbf{x},\mathbf{y})$ denote the Lagrange function of the optimization problem

$$\max_{\mathbf{x}\in\mathbb{R}^n} f(\mathbf{x}) \quad \text{s.t.} \quad g_i(\mathbf{x}) \leq 0 \quad \forall i = 1,\ldots m,$$

where $f, g_1, \ldots, g_m : \mathbb{R}^n \to \mathbb{R}$. Suppose $(\mathbf{x}^*, \mathbf{y}^*)$ is an equilibrium of the two-player game as defined above. Show that $\mathbf{x}^*$ is an optimal solution of the optimization problem.

Exercise 2. Let Γ_a and Γ_b be two zero-sum matrix games which are defined by the following two payoff matrices of the row player (i.e., the first player).

a)

$$\begin{bmatrix} 1 & 0 \\ -2 & 4 \end{bmatrix}$$

b)

$$\begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}$$

Determine the optimal maxmin-strategy for the first player in the corresponding randomized matrix games.

Exercise 3. Let $A = (a_{ij}) \in \mathbb{R}^{m\times n}$ be the payoff matrix of the row player in a zero-sum matrix game. We say that

- row i_1 *dominates* row i_2 if $a_{i_1 j} \geq a_{i_2 j}$ holds for all columns j, and
- column j_1 *dominates* column j_2 if $a_{ij_1} \leq a_{ij_2}$ holds for all rows i.

a) Show that in a randomized matrix game, dominated rows and columns can be ignored when optimal maxmin-strategies are to be calculated.

b) Determine an optimal maxmin-strategy for the row player in the randomized zero-sum matrix game with payoff matrix

$$\begin{bmatrix} 2 & 1 & 1 & 2 \\ 1 & 2 & 0 & 2 \\ 0 & 3 & 4 & 4 \\ 1 & 3 & 5 & 4 \end{bmatrix}.$$

Exercise 4. Consider the network G illustrated in Figure 1 and extend it to a routing game by adding two players with corresponding sources $s_1 = s_2 = s$, sinks $t_1 = t_2 = t$, and demands $d_1 = 1$ and $d_2 = 2$. In an *atomistic routing game*, the players must send the whole demand along a single path, i.e., the strategy set for each of the players is the set of all s,t-paths in G. Show that the game does not admit an equilibrium (in the usual sense, where a profil is an equilibrium if no player would be better off by switching his strategy given that the other players stick to the chosen strategy.)

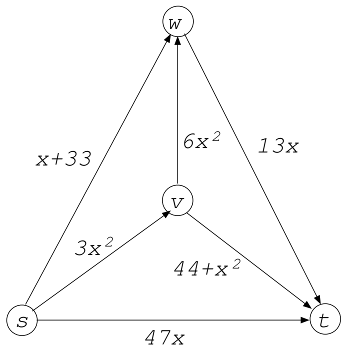

Figure 1: Atomic routing game without equilibrium.