

Assignment 3

Discussion: Thursday, June 13, 2013.

Exercise 1. Consider the facility location game illustrated in Figure 1.

- a) Calculate an allocation vector $x \in \mathbb{R}^3$ in the core.
- b) Extend the graph by adding a new facility such that the core of the modified game has an empty core.

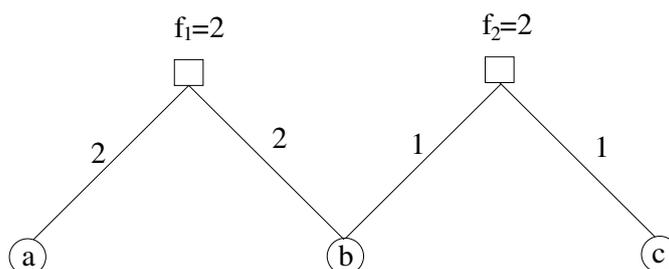


Figure 1: Facility location game with two facilities and three players $N = \{a, b, c\}$.

Exercise 2. Let (N, v) be a cooperative game with $\tilde{v}(N) = v(N)$ and $v(\emptyset) = 0$. We define the corresponding *dual game* by

$$c(S) := v(N) - v(N \setminus S) \quad \forall S \subseteq N.$$

Show that both games have the same core. Moreover, show that v is supermodular if and only if c is submodular.

Exercise 3. Consider the game $\Gamma = (N, v)$ on three players $N = \{1, 2, 3\}$ whose payoff function is defined by $v(\{\}) = v(\{1\}) = v(\{2\}) = v(\{3\}) = 0$, $v(\{1, 2\}) = v(\{2, 3\}) = 1$, $v(\{1, 3\}) = 2$ and $v(\{1, 2, 3\}) = 4$. Determine all marginal vectors and the core of Γ .

Exercise 4. Let $N = \{1, 2, 3\}$ and $v : 2^N \rightarrow \mathbb{R}_+$ be a payoff function with values

$$v(S) = \begin{cases} 0 & \text{if } |S| \leq 1 \\ 60 & \text{if } |S| = 2 \\ 72 & \text{if } |S| = 3. \end{cases}$$

Determine the excesses with respect to the full allocations $x = (30, 30, 12)^T$ and $x' = (24, 24, 24)^T$. Is $l(x)$ lexicographically smaller than $l(x')$? Determine the nucleolus of $\Gamma(N, v)$.

Exercise 5. Consider a parliament with three parties $\{P_1, P_2, P_3\}$, where the parties P_1 and P_2 own 10 seats each, while party P_3 owns 19 seats. A majority of seats is necessary for a coalition to form the regime. How much power has each party? I.e., what is the Shapley value for the game in which each coalition has value 1 if it is able to form the regime, and zero otherwise?