Ship Traffic Optimization for the Kiel Canal

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1 The Kiel Canal

The Kiel Canal connects the North and Baltic seas and is ranked among the world’s three major canals. In fact, in terms of traffic, it is the busiest artificial waterway worldwide. In a billion Euro project, the German Federal Waterways and Shipping Administration plans to enlarge the canal during the coming years. This project is about contributing to a well-founded advise on how the enlargement can be optimally done. In order to evaluate the various construction possibilities it is indispensable to first provide an accurate model for the ship traffic and designing an algorithm which (ideally optimally) controls it. This paper is about such optimal traffic control.

The problem very roughly is as follows. There is bi-directional ship traffic on the canal; there are several locks at both ends. Ships are classified in different size categories. Passing and overtaking is allowed only if the sizes of the two ships do not exceed a given threshold which depends on the meeting point. If otherwise a conflict occurs, ships have to wait at designated, capacitated places, the sidings. The objective is to minimize the total passage time, including lock and siding waiting times. The overall scheduling is currently done by two teams of experienced planners, one for the locks, one for the sidings. In this abstract we concentrate on the latter problem, but both will be treated in an integrated way during the project. Despite significant differences there are certain similarities to train scheduling on a single track line [1].
2 Algorithms for Ship Traffic Control

The canal consists of sidings (set $T$) which alternate with canal segments (set $E$). More precisely, we represent the canal as $t_1, e_1, t_2, e_2, \ldots, e_{|E|}, t_{|T|}$ with sidings $t_i \in T$ and segments $e_i \in E$. We have a set $S$ of $n$ ships, each pair $(s_1, s_2) \in S \times S$ of which may be forbidden to pass each other on a given segment $e \in E$. In this case, we say that $(s_1, s_2)$ has a conflict on $e$ and write $(s_1, s_2) \in C_e$. A ship $s \in S$ needs time $\tau_{s,i} \geq 0$ to pass through a siding or segment $i \in T \cup E$. We formulate a mixed integer program (MIP) which is based on deciding for each ship $s \in S$ when it departs from a siding or segment (this is a natural way of encoding a solution). We have a variable $d_{s,i}$, $i \in T \cup E$ for the respective departure time. For each segment $e \in E$ and each conflicting pair $(s_1, s_2) \in C_e$ of ships we decide which ship will enter $e$ first. This decision is represented by a binary variable $z_{s_1,s_2,e}$ which assumes a value of 1 if and only if $s_1$ enters $e$ before $s_2$ does. Finally, a variable $w_{s,t} \geq 0$ represents the waiting time of ship $s$ in siding $t$. The model reads as follows.

\begin{align}
\text{minimize} & \quad \sum_{s \in S, t \in T} w_{s,t} \\
\text{s.t.} & \quad d_{s,i} + \tau_{s,e_i} = d_{s,e_i} \quad s \in S, i = 1, \ldots, |E| \\
& \quad d_{s,e_i-1} + \tau_{s,t_i} + w_{s,t_i} = d_{s,t_i} \quad s \in S, i = 2, \ldots, |T| \\
& \quad z_{s_1,s_2,e} = 1 \implies d_{s_1,e} \leq d_{s_2,e} - \tau_{s_2,e} \quad e \in E, (s_1, s_2) \in C_e \\
& \quad z_{s_1,s_2,e} = 0 \implies d_{s_2,e} \leq d_{s_1,e} - \tau_{s_1,e} \quad e \in E, (s_1, s_2) \in C_e \\
& \quad d_{s,i} \leq \tilde{d}_{s,i} \quad s \in S, i \in T \cup E \\
& \quad w_{s,t} \geq 0 \quad s \in S, t \in T \\
& \quad z_{s_1,s_2,e} \in \{0, 1\} \quad e \in E, (s_1, s_2) \in C_e
\end{align}

The objective function (1) accounts for minimum total waiting time of all ships, which—when ships travel at their respective full speeds, what is an assumption close enough to reality also for manual planners—is equivalent to minimum total passage time. Constraints (2) and (3) ensure a consistent setting of departure times along each ship’s route. Our notation assumes that all ships travel upstream, that is, in the direction of increasing indices of sidings and segments, but—abusing notation—departure times of downstream ships are set analogously. We avoid the distinction here for the sake of an easier presentation.

In a continuous planning, ships may start and end at any point of the canal, not necessarily only at the ends; thus there are release times for each ship $s \in S$. This implies that constraints (2) and (3) ensure that $d$ variables increase in the travel direction of the ship, but also that they decrease, and even below zero, in the other direction. For this reason, $d$ variables are not restricted in sign, but there are lower and upper bounds $d_{s,i}, \tilde{d}_{s,i}$ imposed on departure times. These bounds are
given in (6). Precedence constraints (4) and (5) link the $d$ and $z$ variables. It is easy to incorporate safety distances between ships as well which in particular avoid that ships are at the same place concurrently. Note that we do not respect siding capacities. In fact, the logic of this formulation assumes that all ships wait at the same point at the end of a siding, no matter how many ships they are. Thus, this MIP is a relaxation of our original problem formulation.

In fact, to overcome this inaccuracy one needs some sort of time and space discretization. In particular the latter one causes some difficulties due to lots of different ship sizes and arbitrary start and end points. Instead of respecting such a discretization in the model directly we chose to let the algorithm take care of that. We designed a successive shortest path algorithm respecting blocked time windows which constructs conflict-free dynamic routes for the ships one after another. In addition to the basic version described in [2] our algorithm allows more than one ship to be on an edge, handles arbitrary start and end positions, and tries different valid waiting positions in a siding. Thus, an initial solution is constructed which is feasible with respect to all constraints (also those not mentioned here). The running time is a few seconds. A local search based on a loss-benefit calculation improves that first solution in a rolling horizon manner, mildly imitating a manual planner’s procedure. Since this local search uses the dynamic routing algorithm as a subroutine also the final schedule respects all the constraints. Our solutions are visualized in exactly the same way the planners are used to see them, in a way-time diagram of the canal, see Figure 1.

3 Lower Bounds

Not only for theoretical reasons we want to assess the quality of the heuristic solutions we obtain. The above MIP is suited for this purpose, however, for practical problem instances it takes much too long to solve it to integer optimality, mainly because of the poor LP relaxation. Instead, we developed a much more involved model which is solved by a full-fledged branch-and-price algorithm. The idea is to base an integer program on schedules for all ships on a given segment. Via a sort of flow conservation constraints these schedules for segments are linked together to form a valid schedule for the entire canal. The pricing subproblem is a scheduling problem which is interesting in its own right. In principle, this model is suited to seamlessly integrate the lock scheduling problem as well. To price the corresponding variables, a particular two-dimensional packing problem needs to be solved.

4 Preliminary Results and Perspectives

We are provided with historical ship traffic data from recent years and forecasts for the year 2025. This way, we can compare our schedules to the manual plans. We refrain from giving numerical
results here. However, in an intermediate evaluation the officers in charge of the enlargement were impressed and satisfied by the possibilities operations research and discrete optimization have to offer. We were encouraged to extend our models to take all further details of the planning situation into account, in particular the locks at both ends of the canal.

Figure 1: Example of a way-time diagram showing one of our solutions.

References
